INVESTIGATION OF SANDS
SUBJECTED TO DYNAMIC LOADING

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PREFACE

The information contained herein was developed on Research study 2-5-62-33 entitled "Piling Behavior" which is a cooperative research study sponsored jointly by the Texas Highway Department and the U. S. Department of Transportation, Federal Highway Administration, Bureau of Public Roads. The broad objective of this project is to fully develop the use of the computer solution of the wave equation so that it may be used to predict driving stresses in piling and to estimate static load bearing capacity of piling from driving resistance records.

This report should be considered as a supplement of Research Report Number 33-7. As stated in the preface, Research Report Number 33-7 was considered as an interim report of an exploratory investigation into the dynamic load-deformation properties of sands. This supplement (Research Report 33-7A) includes improvements in instrumentation which were made since the interim report was published, and it presents results of final tests on the dynamic properties of three different sands.

Peak dynamic and static strengths of saturated sand samples under triaxial confinement are determined experimentally and presented herein. Particular attention is given to the effects of loading velocity, sample density, and intergranular pressure. Experimental results are compared with results predicted by using the rheological model currently in use with the computer solution of the wave equation. A modification of this rheological model is proposed so that it can be used to reproduce experimental data and correctly predict peak dynamic load.

The opinions, findings, and conclusions expressed in this report are those of the authors and not necessarily those of the Bureau of Public Roads.
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NOTATION

c = A viscous damping constant for soil.
D = Displacement of pile mass element in time interval n, in inches.
D' = Ground plastic displacement in interval n, in inches.
d = Displacement of pile mass element in time interval n-1, in inches.
d' = Displacement of pile mass element in time interval n-2, in inches.
e = Void ratio.
F_r = The total movement resistant force of the rheological model, in pounds.
g = Acceleration due to gravity, in feet per second per second.
I = Empirical constant, the intercept found in Figure 10.
J = A damping factor in the rheological model applied at the tip of the pile, in seconds per foot.
J' = A damping factor in the rheological model for use as skin friction, in seconds per foot.
K = Spring constant for pile mass segment, in pounds per inch.
K' = Spring constant for soil mass segment, in pounds per inch.
m = Subscript denoting general case.
p = Subscript denoting point of pile.
P_{dyna} = Dynamic strength of soil, in pounds.
P_{stat} = Static strength of soil, in pounds.
R = Resistance in time interval n, in pounds.
\Delta t = Time interval used for calculation, in seconds.
V = Loading velocity, in feet per second.
W = Weight of pile segment, in pounds.
\bar{\sigma} = Effective confining pressure, in pounds per square inch.
in = Inch or inches.
ipm = Inches per minute.
ft = Foot or feet.
lb = Pound or pounds.
psi = Pounds per square inch.
fps = Feet per second.
Investigation of Sands Subjected to Dynamic Loading

Chapter I
INTRODUCTION

Dynamic Loads and the Behavior of Soil

The increase of strength in soils with rate of loading has been observed for several years. This phenomenon is apparent in unconfined as well as triaxially confined test specimens. Other investigators have worked with rapid loading in triaxial tests, and have noted the effects of such variables as time rate of loading, void ratio, and intergranular pressure on the peak strength of the test samples.

A mathematical model was developed to analyze the dynamic behavior of bearing piles. This model was presented by E. A. L. Smith and augmented by C. H. Samson, T. J. Hirsch, and L. L. Lowery. Attention is given to the soil damping constants employed, particularly at the pile tip. Smith's model is that of a segmented pile with each segment of mass in the pile being displaced in a given time interval by a distance

\[ D_m = 2d_m - d'_m + \frac{12g\Delta t^2}{W_m} \]  

where

\[ D = \text{Displacement of pile mass segment in time interval } n, \text{ in inches}; \]
\[ d = \text{Displacement of pile mass segment in time interval } n - 1, \text{ in inches}; \]
\[ d' = \text{Displacement of pile mass segment in time interval } n - 2, \text{ in inches}; \]
\[ g = \text{Acceleration due to gravity, in feet per second per second}; \]
\[ K = \text{Spring constant of pile mass segment, in pounds per inch}; \]
\[ m = \text{Subscript denoting general case}; \]
\[ R = \text{Resistance to movement in time interval } n, \text{ in pounds}; \]
\[ \Delta t = \text{Time interval used for calculations, in seconds}; \]
\[ W = \text{Weight of pile segment, in pounds}. \]

The soil frictional resistance along the sides of the pile is described by the equation

\[ R_m = (D_m - D'_m) K'_m (1 + J'V_m) \]  

where

\[ p = \text{Subscript denoting point of pile}; \]
\[ J = \text{Damping constant of soil at point of pile}. \]

Since the maximum value of \((D_p - D'_p) K'_p\) is the static strength of the soil, then Equation 3 can be written more simply as

\[ P_{\text{dynamic}} = P_{\text{static}} (1 + J V) \]  

where

\[ V = \text{Impact velocity of dynamic load}; \]
\[ P_{\text{dynamic}} = \text{Peak load developed in dynamically loaded sample loaded at a constant velocity } V; \]
\[ P_{\text{static}} = \text{Peak load developed in statically loaded sample}; \]
\[ J = \text{A damping constant}. \]

If \(P_{\text{dynamic}}, P_{\text{static}}, \text{and } V\) are measured in the laboratory, then \(J\) can be calculated by

\[ J = \frac{1}{V} \left[ \left( \frac{P_{\text{dynamic}}}{P_{\text{static}}} \right) - 1 \right] \]  

Smith suggested that an empirical value for \(J\) be used in pile driving analysis. This empirical value was based on an arbitrary relationship between the viscous damping along the sides of the pile and at the tip. At present the total effect of the damping is based on experience and observation. As this model is employed, it is assumed that \(J\) is constant within a set of initial conditions, and that the increase in the soil's strength with rate of loading is encompassed by the increase of velocity, \(V\), in the above equations.

Objectives

In this research an experimental investigation was made to evaluate the validity of Equation 4 and the assumptions made in employing it. To accomplish this objective, the work plan given below was followed.

1. The peak static and dynamic strengths of three sands were determined experimentally using different rates of loading, void ratios, and intergranular pressures.
2. Experimental results were compared with values predicted by Smith's proposed rheological model.
3. A study of Smith's model was undertaken and modifications were proposed in order to obtain better agreement with experimental results.

*Numbers indicate references on Page 8.
The authors are hesitant to say that the laboratory model using triaxial confinement (as later described) actually simulates the soil failure at the tip of a pile being driven in the field. However, by varying the initial conditions and by testing different types of sand, an effort is made to define general trends of the dynamic to static strength ratio under various situations. By these general trends the behavior of \( J \) may be better described and Equations 2 and 3 may be more meaningful.

Chapter II
INSTRUMENTATION

Equipment Used

Loading sand samples dynamically in triaxial confinement has been done by other investigators.\(^1\),\(^2\),\(^3\) In this research some of their procedures are used with modifications. The main difference, however, is in the loading machine (see Figure 1), and in the load cell beneath the sample (see Figure 2). The method of sample preparation is similar to that outlined by Whitman and Healy\(^3\) in that the sand is spooned into a triaxial mold in a presaturated state.

The loading rates of interest in this research were between 3 fps and 12 fps. This range covered the pile velocities achieved during driving with most commercial pile driving hammers. It was decided that the impact velocity and force required could best be obtained from a free falling mass. Since Smith’s equation can be written in terms of peak dynamic strength, the falling mass should be sufficient to develop the peak strength in every test. Preliminary testing was done to find the minimum weight required. Dense standard triaxial samples were made from the sand with the highest angle of internal friction. A weight was then found (by trial and error) that would obviously fail the dense standard sample when dropped from a height to give the lowest loading velocity of interest (3 fps). It was reasoned that the weight obtained in this manner was sufficient to develop the required peak load in any of the samples to be tested. The weight selected was 165 pounds.

All measuring devices were connected to a Honeywell Type 119 Amplifier and a Honeywell Type 1508 Visicorder. The visicorder output data were recorded by ultraviolet light traces on Kodak linograph direct print paper, exposed at a specified paper speed (see Figure 4).

The supporting frame and table of the loading machine were designed so that the stress waves created by stopping the falling weight would not interfere with the load measurement from the soil sample. The arrangement and structure of the frame allowed the strain gauges, in the load cell at the bottom of the sample, to sense the peak load transmitted by the sample before the stress wave created by stopping the falling weight arrived. It is felt that this endeavor was successful.

It was found that best results were obtained on the oscillograph trace when the striker head on the bottom...
of the falling weight was covered with a 1/8 inch neoprene rubber pad. More cushioning than this caused an irregular trace from the displacement transducer. It was feared that the machinery would be damaged if the cushioning pad had been omitted entirely.

As can be seen in Figure 1, the weight can be raised to any height above the shaft from the triaxial cell by means of a hand crank. The weight is then released by a pull switch, and falls to impact on the shaft. The weight supporting stem passes through a roller bearing guide hole in the guide frame.

The load cell, upon which the sand sample is initially prepared (see Figures 2 and 3), consists of a standard triaxial base with a modified pedestal. This hollow pedestal was built with four strain gauges inside, one mounted at each ninety degrees around the inside circumference. Three drilled holes were made in the top plate of the pedestal so that direct connections could be made between valves outside the triaxial cell and the porous stone at the bottom of the sample. A similar connection was made to the top cap on the sample. Through these valves and connections, pore pressures were regulated for the desired initial conditions (depending on the lateral confining air pressure in the cell) and were measured throughout the test (see Figure 4).

The vertical deformation of the sample was measured by attaching a linear displacement transducer to the piston at the top of the triaxial cell. This arrangement (see Figure 3) measured the movement of the loading shaft into the triaxial cell, or the amount of axial deformation of the entire sample. The slope of the oscillograph trace of this movement also yielded the time rate of loading (V) of the sample.

It should be noted that the rate of sample deformation measured from the oscillograph trace was always slightly less than the impact velocity of the mass calculated on the basis of a free fall from a known height.

(SEE APPENDIX "A" FOR INTERPRETATION)

Figure 4. Visicorder trace for a typical test.
This is believed to be the result of friction in the roller bearing guide hole on the weight supporting stem, and, possibly, of the damping action of the entire system at impact.

Static tests were done using a model AP-322-X Soiltest compression machine. In these tests a calibrated proving ring was used in series with the load cell to check the values of peak load obtained. The static tests were run at a loading rate of 0.05 ipm.

The operation and calibration of all commercially made equipment was done in accordance with operation manuals prepared by the manufacturers.

**Calibration**

Initially the load cell shown in Figure 2 was made as one piece, but the calibration curve obtained in the manner later described was not linear. After some investigation, it was reasoned that the one piece construction caused a moment transfer from the horizontal top surface of the pedestal to the vertical walls where the strain gauges were mounted. This created a situation similar to an indeterminant frame. To rectify the situation, the horizontal plate was cut free so that no moment could be transferred to the strain gauged surface.

Calibration of the load cell was done by mounting a proving ring on the top of the load cell and subjecting them both in series to various load increments in a standard Soiltest compression machine. While the cell was being loaded, an attenuation and range were selected on the amplifier which gave a convenient scale for load readings on the visicorder trace.

The pressure transducers used to monitor pore pressures were Type 4-312-0001, manufactured by Consolidated Electrodynamic Corporation. These transducers were calibrated by mounting them on the triaxial cell with values open to the confinement pressure (see Figure 3). Using a calibrated pressure gauge and various pressure increments, convenient scales were chosen for the output trace in a manner similar to that used for the load cell.

A linear displacement transducer (Model 7DCDT-100 linear motion potentiometer made by the Sanborn Company) was used to measure the movement of the loading piston into the triaxial cell. This transducer consists of a variable resistor controlled by a plunger which is free to move up or down inside an outer casing.

To calibrate the transducer, the casing was clamped into place, and the plunger was attached to a threaded screw which, in turn, was attached to an Ames dial. This mechanism allowed the movement of the plunger to be directly measured as the screw was turned. A convenient scale was chosen on the visicorder trace by means of an adjustable resistance, so that the actual movement of the plunger, as indicated by the Ames dial, was correlated to the movement of the ultraviolet light spot on the visicorder.

All the calibrations were checked during the course of testing. It was found that the load cell and pressure transducers calibrations did not vary significantly, but the calibration of the displacement transducer needed to be re-established before each test.
Chapter III

DYNAMIC TESTS ON OTTAWA SAND

Sample Preparation

Saturated samples of 20-30 Ottawa sand at two void ratios (0.50 and 0.60) were prepared. The procedure is basically the spooning of wet, presaturated sand into water held in a rubber membrane and metal forming jacket. The samples were 2.84 inches in diameter and 6.0 inches high. By calculating a dry weight of sand, based on the volume of the sample mold, and then soaking the dry sand in desired-distilled water before placing the sand in the mold, the desired void ratios could be obtained.

Much care was exercised in order to obtain a highly saturated sample, and a considerable amount of practice was necessary to develop the techniques needed to prepare a saturated sample at a desired void ratio.

Initial Conditions

In order to define the basic trends of the viscous damping constant, various sets of initial conditions were utilized in the tests on Ottawa sand.

A lateral confining pressure of 30 psi was used so that a rather large range of pore pressures could be examined. The parameter of effective confining pressure (intergranular pressure) was explored by comparing drained tests with undrained tests. In an undrained test the pore pressure would be 28 psi, and intergranular pressure would be two. In a drained test the pore pressure would be zero (or atmospheric) and the intergranular pressure would be 30 psi.

In order to describe each set of parameters properly, it was felt that at least 14 tests should be run. Several additional tests were run to substantiate the data obtained. In general, test results could be duplicated within five percent agreement.

Since Smith's mathematical model (see Equation 5) uses the dynamic strength of the soil as a ratio of dynamic peak strength to static peak strength, static tests were also performed. It was reasoned that excess pore water pressure in sand would have ample time to dissipate under a static loading condition in the field. Therefore, all static tests used in the above mentioned ratio are drained, with the pore pressure inside the sample maintained at zero.

It should be noted that when the samples were drained before testing, from 1 to 2 cc of water was forced out of the sample and the drainage control system. It was observed, however, that the sample height did not change, and it is believed that the water came from the rough surface of the sample as the membrane was drawn more tightly to the surface. It is not believed that the particle structure was significantly altered by the draining process.

Results of the Test Series

A summary of the dynamic test series is shown in Table 1. These results are also plotted in Figures 6 through 8. From the sample visicorder trace (Figure 4) it is seen that the sample sustains its peak load for about 0.01 second, and then fails. The movement of the loading shaft is constant throughout this time, and the pore pressure goes negative, after a slight increase at the instant of impact. This pore pressure decrease is felt to be much more rapid within the sample, but due to the pressure transducer's location away from the base of the sample, the delay is understandable.

Figure 6 shows that the peak dynamic strength is not greatly increased by increasing the loading velocity if, (1) the sample is dense, or (2) the intergranular pressure (effective confining pressure) is sufficiently high. The ratio of dynamic to static strength in Figure 7 shows a similar trend. If it is reasoned that a saturated sand under a typical dynamic load in the category 3 situation (discussed in Chapter I) does not have time to dissipate its excess pore water pressure before failure, then the peak dynamic strength would depend on the density of the sand and the rate of loading. For the case of a saturated sand under the tip of a pile, it may be assumed that the sand has been densified by previous pile penetration and resultant vibrations, if not already dense in its natural state.

Table 1. Test Series Results—20-30 Ottawa Sand

<table>
<thead>
<tr>
<th>Initial Pore Pressure (psi)</th>
<th>V (fps)</th>
<th>P_{dynamic} (lbs.)</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>4.17</td>
<td>723</td>
<td>1.18</td>
</tr>
<tr>
<td>28</td>
<td>6.80</td>
<td>750</td>
<td>1.21</td>
</tr>
<tr>
<td>28</td>
<td>9.50</td>
<td>763</td>
<td>1.23</td>
</tr>
<tr>
<td>0</td>
<td>2.94</td>
<td>763</td>
<td>1.23</td>
</tr>
<tr>
<td>0</td>
<td>7.00</td>
<td>770</td>
<td>1.24</td>
</tr>
<tr>
<td>0</td>
<td>11.74</td>
<td>788</td>
<td>1.27</td>
</tr>
<tr>
<td>e = 0.60</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Pore Pressure (psi)</th>
<th>V (fps)</th>
<th>P_{dynamic} (lbs.)</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>4.83</td>
<td>441</td>
<td>0.95</td>
</tr>
<tr>
<td>28</td>
<td>8.87</td>
<td>510</td>
<td>1.09</td>
</tr>
<tr>
<td>28</td>
<td>12.75</td>
<td>593</td>
<td>1.26</td>
</tr>
<tr>
<td>0</td>
<td>3.53</td>
<td>583</td>
<td>1.26</td>
</tr>
<tr>
<td>0</td>
<td>8.06</td>
<td>620</td>
<td>1.34</td>
</tr>
<tr>
<td>0</td>
<td>12.75</td>
<td>640</td>
<td>1.38</td>
</tr>
</tbody>
</table>
Figure 7. Dynamic to static strength ratio versus loading velocity for Ottawa sand.

For this case, the peak dynamic strength and the dynamic strength to static strength ratio increase only slightly with increased loading rate. This is significant in that the viscous damping constant employed in Smith’s equation is assumed to be constant within a set of initial conditions, and that an increase in the soil’s strength with rate of loading is encompassed by an increase of \( V \) in Equation 4. That is, the curves in Figures 6 and 7 are assumed to have a greater slope than was found experimentally and the curves in Figure 8 are assumed to be straight and horizontal.

Chapter IV
TEST PROGRAM ON OTHER SANDS

Victoria Sand

Since this test series was intended to simulate the actual field conditions, a lateral confining pressure of 15 psi was used. This approximately corresponded to the confining pressure at 16 feet below the surface of the ground at the test site. Perhaps a higher confining pressure existed as the pile was being driven, but the Ottawa sand tests indicated that this increase would not greatly alter the slope of the peak load versus loading velocity curve, if the sand was dense.

A void ratio of 0.56 was used, based on the insitu void ratio and a reasonable densifying effect of the pile driving operation. The procedure for the tests was identical to that outlined for Ottawa sand. A summary of the results obtained from the Victoria sand tests is shown in Table II.

<table>
<thead>
<tr>
<th>Table II. Test Series Results—Victoria Sand</th>
</tr>
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<tbody>
<tr>
<td>( e = 0.56 )</td>
</tr>
<tr>
<td>Confining Pressure = 15 psi</td>
</tr>
<tr>
<td>14.5</td>
</tr>
<tr>
<td>14.5</td>
</tr>
<tr>
<td>14.5</td>
</tr>
</tbody>
</table>

Without plotting the results shown, it can be seen that the same trends are present as those found using Ottawa sand. The damping constant, \( J \), decreases with an increase in \( V \) and, therefore, is not exactly as Smith’s equation describes.

Arkansas Sand

As in Victoria sand, the initial conditions chosen were intended to simulate the field conditions. This particular sand was taken from the site of a lock and dam in Arkansas and has been studied to some extent.

The same sample preparation and testing techniques were employed as previously described, with the only exceptions being those necessary to make the initial conditions similar to those in the insitu state.

The results of the test series are shown in Table III. Again the same trends were found as in Ottawa and Victoria sands.

<table>
<thead>
<tr>
<th>Table III. Test Series Results—Arkansas Sand</th>
</tr>
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<tbody>
<tr>
<td>( e = 0.55 )</td>
</tr>
<tr>
<td>Confining Pressure = 15 psi</td>
</tr>
<tr>
<td>14.5</td>
</tr>
<tr>
<td>14.5</td>
</tr>
<tr>
<td>14.5</td>
</tr>
</tbody>
</table>
Chapter V
CORRELATION, CONCLUSIONS AND RECOMMENDATIONS

Correlation of Laboratory and Field Data

It was found that the values for J, employed in the computer studies of field tests on Victoria and Arkansas sands, varied from 0.00 to 0.30. These values were assigned, based on approximations, to yield the best computer analysis corresponding to the pile load tests done in the field.

While the values of J obtained in the previous chapter are within these limits, they vary significantly with a change in V. As previously mentioned, J should remain constant in Smith's model as V changes, therefore a further study of the rheological model was undertaken.

Discussion of the Rheological Model

In order to modify Equation 4 so that J is constant within a set of initial conditions, an examination of the Smith model is necessary. It should be noted again that only Equation 4 of Smith's solution is questioned here.

Figure 9 shows the rheological model proposed by E. A. L. Smith. The pile-soil system is simulated by a spring and sliding block in series connected to a dashpot in parallel. From this arrangement the equation for the resisting force of the system is

\[ F_r = K'x + cx \]  

where

- \( F_r \) = Resisting force of the system;
- \( K' \) = A spring constant for the soil;
- \( c \) = A viscous damping constant for the soil;
- \( x \) = The distance the soil deforms elastically, i.e., the quake; and
- \( \dot{x} \) = The velocity of soil deformation.

In order to incorporate the effects of size and shape of the pile, Smith has made

\[ c = K'xJ \]  

where J is some damping constant that is intended to perform a similar function as c.

With this relation, the maximum \( F_r \) becomes \( F_{\text{dynamic}} \) and, if the maximum value of \( K'x \) is the static strength of the soil, Equation 6 becomes Equation 4.

This model is logically employed, but, as is often the case, the experimental data collected are not adequately described by the theoretical prediction. To best utilize the Smith model and the data obtained in this research, it is felt that a purely empirical modification might be made on Equation 4 which would allow it to effectively describe the data. No mathematical proof is offered for the modification except for the fact that these experimental results will be described.

Empirical Modification

Several approaches to modifying Equation 4 were tried and discarded. The key to the method found most desirable is the finding that in all the sands tested, the dynamic strength of a dense sand sample at loading velocities from 3 fps to 12 fps forms a plateau with only a slight slope, but decidedly above the static strength (see Figure 10). If J is, in fact, the slope of this plateau, then the intercept of the extended plateau and the ordinate (\( F_{\text{dynamic}}/F_{\text{static}} \)) should also be evaluated empirically.

The authors found that by determining the intercept, I, in the manner shown in Figure 10, and employing it in Equation 4 as shown below, the data were well correlated.

\[ F_{\text{dynamic}} = F_{\text{static}} (I + J V) \]  

Using this relation and the experimental data, J is found to be acceptably constant within a set of initial conditions, with a variable V. All the experimental

![Figure 9. Smith's rheological model.](image)

![Figure 10. Method of evaluating J.](image)
TABLE IV. TEST SERIES RESULTS USING EQUATION 8

<table>
<thead>
<tr>
<th>Initial Confining Pressure (psi)</th>
<th>P\textsubscript{dynamic} (lbs.)</th>
<th>P\textsubscript{static}</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa Sand — e = 0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>733</td>
<td>1.18</td>
<td>1.14</td>
<td>0.009</td>
</tr>
<tr>
<td>30</td>
<td>745</td>
<td>1.20</td>
<td>1.14</td>
<td>0.008</td>
</tr>
<tr>
<td>30</td>
<td>763</td>
<td>1.23</td>
<td>1.14</td>
<td>0.009</td>
</tr>
<tr>
<td>Arkansas Sand — e = 0.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>658</td>
<td>1.29</td>
<td>1.27</td>
<td>0.006</td>
</tr>
<tr>
<td>15</td>
<td>665</td>
<td>1.30</td>
<td>1.27</td>
<td>0.006</td>
</tr>
<tr>
<td>15</td>
<td>673</td>
<td>1.32</td>
<td>1.27</td>
<td>0.006</td>
</tr>
<tr>
<td>Victoria Sand — e = 0.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>762</td>
<td>1.51</td>
<td>1.49</td>
<td>0.006</td>
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<tr>
<td>15</td>
<td>774</td>
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<tr>
<td>15</td>
<td>778</td>
<td>1.54</td>
<td>1.49</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Results of the sands previously tested which are thought to be likely for the condition under a pile being driven (as discussed in Chapter III) are shown in Table IV, with J calculated from Equation 8.

**Recommendations**

It should be noted that the range of loading velocities in this research was limited. Only in the range of 3 fps to 12 fps can the foregoing modification to Smith’s equation be suggested. Also, only relatively dense sands were of particular interest. These two limitations are sufficient to describe the assumed behavior of a pile-sand system, and therefore, some interesting phenomena which were observed during the course of testing were not vigorously explored. A further study of dense and loose sand subjected to dynamic loading over a wide range of loading velocities is particularly recommended.

Several more questions and problems were introduced by this research than were considered before, but if the empirical suggestions herein can give computer solutions of Smith’s wave equation more reliability, then the research was successful.

**References**


Appendix

DATA INTERPRETATION

The Visicorder Trace

The sample visicorder trace, shown in Figure 1A, is presented to explain the interpretation of the test data. This trace is a reduced scale drawing of the actual visicorder trace from a dynamic test on a Victoria sand sample. The sample had a void ratio of 0.56 and was in a triaxial confinement pressure of 15 psi ($V = 8.58$ fps). Intergranular pressure was 0.5 and no drainage was allowed.

In this test, the paper speed was 120 ips and the vertical lines placed uniformly on the trace are at 0.01 second intervals. The following comments correspond to points on the sample trace.

1. Trace from load cell. The predetermined calibration is 1 inch = 710 pounds and downward movement indicates compression. At this point, the galvanometer has been balanced to indicate no load on the cell.

2. The falling weight has begun to load the sample. Inertial properties of the reflector mirrors in the galvanometer bank of the visicorder produce the smooth curve at this point.

3. The peak load is reached and the sample sustains this load for something less than 0.01 second.

4. After the sample has failed, the load returns to zero.

5. The trace from a pressure transducer. Calibration is 1 inch = 10 psi and a positive pressure of 14.5 psi is indicated inside the sample. The direction of negative pressure is upward.

6. At the instant of impact, a slight positive pressure is registered, but as the sample expands under the load, the pressure inside the sample goes negative.

7. After failure, the pressure inside the sample is a negative one atmosphere.

8. The trace from the linear displacement transducer indicates vertical reduction in height of the sample. Calibration is 1 inch = 0.1 in actual movement, and the trace moves upward as the sample is shortened.

9. The sample is deformed by the falling weight.

10. Here the sample is deformed at a constant rate. The slope of this straight portion is the rate of failure, or "$V$" in Equation 4.

11. The weight has rebounded from the catch frame and the transducer is pushed upward by the confinement pressure of the triaxial cell.

Figure 1A. Sample visicorder trace.