A COMPUTER TECHNIQUE FOR PERSPECTIVE PLOTTING OF ROADWAYS

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INTRODUCTION

In conventional design procedures, the highway designer is dependent upon his ability to envision the roadway in perspective based on plan and profile views of the roadway. Considering the complexity of our modern highways, it is truly a formidable task to visualize the roadway in complete detail. Invariably there are details which would improve the safety and efficiency of operation substantially if they had been detected at the design stage. For this reason, it is highly desirable that some means be developed whereby the designer may view his design prior to construction. Models of complex interchanges have been constructed to satisfy this need, but models are expensive and generally cannot be justified for extensive application. For the less complex design applications, the perspective drawing can be an effective alternate. However, several perspectives on each approach to a highway feature could be expensive and time consuming unless manual means of drawing were replaced by more modern techniques.

In this investigation, the digital computer has been used in the development of a means of preparing perspective views of a roadway. The technique developed herein would conceivably have application in general roadway design, interchange design, and in the location of signs and other traffic control devices for greatest effectiveness. An attempt has been made to provide a workable tool based on sound principles which can be easily understood and used by designers having little or no previous computer knowledge. The numerical methods of this research were developed and checked on the IBM 7094 digital computer. The program package was organized as a 7094 FORTRAN main program which links several 7094 sub-routines to perform the various manipulations required. A Calcomp model 565 digital plotter linked to an IBM 1401 digital computer was used to draw the individual roadway plots.

BACKGROUND

Several approaches have been taken in studying driver reaction as related to the driving task, and to traffic operations in general. In most cases, these investigations have dealt only with single elements of the roadway or traffic stream in relation to driver reaction or behavior.
and traffic operation. Recently however, increasing emphasis has been placed on the driver's view of the roadway or the total vision input as related to the driving task.

Various mechanical devices have been employed in studying the driver's reaction in relation to his visual input. Counters and measuring devices have been used on test drivers to determine driving patterns over specifically designated test roadways. One such study was conducted at the Transportation Institute at the University of Michigan. The driving patterns of more than 950 drivers were observed over a selected 5 1/2 mile route. Measuring devices yielded information concerning stopping time, acceleration actions, steering reversals, brake applications, speed changes, and direction changes. Photographic equipment mounted in the test vehicle was used to record the visual input of the driver. These visual inputs were classified into two classes: those relating to the driving task, and those unrelated. An unrelated event was defined as one that had no potential for requiring the driver to change the motions of the vehicle.

An interesting method of studying the isolation of visual input was investigated by Gordon. An apparatus was developed to be worn by the test driver which restricted his vision and recorded his visual fixation points. The apparatus, an aperture observation tube equipped with an 8 mm camera and mounted on a head helmet, can be adjusted to allow various angles of vision. It was found that all drivers guide their vehicles by reference to the road edges and the center line. Two factors which seem to limit this method are: (a) the accuracy of questionnaires answered by test drivers for indicating the driver's visual input, and (b) the limiting effect imposed by the small aperture of the head apparatus. Gordon reported that the driver should always be given an unimpeded view ahead sufficient to satisfy his anticipation requirements. Further research is needed in the field of perceptual requirements in the many situations encountered on the roadway.

Finally, the use of simulation has become effective in the study of driver behavior. The simulation must always strive to completely represent the visual, auditory, and sensing stimuli to which the driver is exposed. Full-scale simulators are still a thing of the future but part-time simulators have been developed to simulate particular parts of the driving task. The part-time simulator provides an ideal laboratory that allows flexibility and modification to meet the changing
requirements and insights to a research program. The Institute of Transportation and Traffic Engineering at U. C. L. A. has been conducting research for a number of years using a driving simulation laboratory. An operating vehicle mounted on a chassis dynamometer was utilized with motion picture films to give the operator actual driving sensation. Both speed and steering wheel movements indicated that drivers responded appropriately to different roadway and environmental conditions.

It is felt that the plotting scheme of this investigation can very effectively supplement existing techniques in this field. An accurate picture of the driver's visual input is needed as a basis for design decisions. Gordon has indicated that when man's input has been specified, driving itself will be, to a considerable extent, described.²

GENERAL CONCEPTS

The basic concepts involved in the study are fundamentals of elementary geometry and highway engineering principles. The description of the roadway in perspective as the driver sees it is produced by projecting selected points from a three dimensional roadway onto a common two dimensional reference plane as shown in Figure 1. The selected points, represented by X and Y coordinates on a two dimensional reference plane are joined to represent the driver's visual input. Although the underlying principles of the perspective plotting technique are familiar to all highway engineers, it is considered desirable at this point to review these principles in order to establish the basic concepts of this study.

VERTICAL ALIGNMENT

The common parabolic vertical curve with length controlled by minimum stopping sight distance is used in this study. For computational purposes the rate of vertical curvature, which is based on stopping sight distance in relation to the height of object, is defined by K, the horizontal distance in feet required to effect a one percent change in gradient. As an example, the normal rate of curvature, K, for a 70 mph design speed is 257 for crest vertical curves and 145 for sag vertical curves.
GENERAL CONCEPT OF PERSPECTIVE PLOTTING PROGRAM

FIGURE 1
The length of vertical curve can be calculated using the relation

\[ L = KA \]  \hspace{1cm} (1)

where \( L \) is the length of the vertical curve, \( K \) is the design control for curvature, and \( A \) is the absolute algebraic difference in grades.

In Figure 2 (a), \( G_1 \) and \( G_2 \) represent the grades of the tangent sections on each vertical curve. The station and elevation of the beginning of the curve and the station of the end of the curve are now accessible. The offset from the gradient to the curve can be computed using the characteristic of the parabola that the offset from a parabola to its tangent varies as the square of the distance from the point of tangency.

\[ \text{OFFSET} = \frac{(G_1 - G_2) X^2}{2L} \]  \hspace{1cm} (2)

where \( X \) is the distance to the station point in 100-foot stations, and \( L \) is the horizontal length of the curve in 100-foot stations. The equation of the curve then becomes

\[ Y = E + \frac{G_1}{100} X - \frac{G_1 - G_2}{2L (100)} X^2 \]  \hspace{1cm} (3)

where \( E \) is now the elevation of the Vertical Point of Curvature (VPC) and \( Y \) is the elevation of the station point in feet. Again, \( X \) is the distance to the station point in 100-foot stations. The slope at any station point on the roadway is obtained from the first derivative of equation (3) with respect to \( X \).

\[ \frac{dy}{dx} = \frac{G_1 - G_2}{100 - 100 (L) X} \]  \hspace{1cm} (4)

Since the driver's eye is assumed to be parallel with the roadway directly beneath his vehicle, the angle between the driver's eye and the horizontal is given by

\[ \theta = \arctan \frac{dy}{dx} \]  \hspace{1cm} (5)

When the VPC lies beyond the VPT of the preceding curve, the slope is known to be constant and equal in magnitude to the slope at the prior VPT. The elevation of the station points must be calculated using points defined earlier. The distance in feet may be
calculated between the two tangency points and, from the tangent grade of the preceding curve, the total vertical change in elevation may be calculated. After determining the rate of change of elevation, the elevations of the station points are readily accessible.

**HORIZONTAL ALIGNMENT**

To describe the horizontal alignment of a roadway so that it may be used in the perspective plotting technique, it is necessary to describe all tangent and curve sections in relation to an assumed reference plane. This is done in the same manner and for the same reason that all vertical curves and tangents are referenced to a horizontal line in conventional design practice. An illustration of relating a horizontal curve to the assumed reference plane is shown in Figure 2b. A-A represents the assumed reference plane, and B-PC is the horizontal distance from the reference plane to the point of curve. Since the radius of the circular curve is perpendicular to the tangent line, $\alpha_1$, the angle between the radius of the curve and the horizontal line B-PC, is equal to $\alpha_2$, the angle formed by the tangent line and a parallel to the reference plane A-A. Line C-C which is the distance from the reference plane to the center of the curve is calculated as

$$C-C = B \pm (R) \cos |\alpha_1| \quad (6)$$

with the negative sign applying if the curve is to the left as in Figure 2b or with the positive sign applying if the curve is to the right. The angle $\beta$ is termed the reference angle and is obtained as the complement of angle $\alpha_1$. By successive additions to the reference angle, the horizontal distance from the reference plane to all station points may be calculated.

**SIGHT DISTANCES**

For this study minimum sight distances are based on stopping sight distance where the driver's eye is considered to be 3.75 feet above the surface of the roadway. On crest vertical curves the sight distance is limited by a point on the roadway surface while on horizontal curves it is limited by lateral obstructions along the roadside.
2a TYPICAL VERTICAL CURVE

2b TYPICAL HORIZONTAL CURVE

HORIZONTAL AND VERTICAL CURVES

FIGURE 2
PERSPECTIVE CONCEPTS

In order to reproduce accurately the driver's image of the roadway, it is necessary to translate the individual station points to the driver's eye and measure their relative perspective distances on a common reference surface. By translating both the horizontal and vertical measurements of a station point, a new set of data points is derived and these may be plotted to give an accurate roadway image.

A common reference line is arbitrarily chosen to be perpendicular to the driver's line of vision and a constant distance in front of the driver's eye. In Figure 3a, A-A represents the reference plane, and B-B represents the reference line, termed perspective reference line, that is perpendicular to the path of the vehicle; C-C represents the vision line of the driver. Lines to successive station points are drawn back across the perspective reference line to the driver's eyes and a perspective horizontal value is derived by measuring the distance between the point and the driver's vision line. A perspective vertical value can be calculated by measuring the elevation of the projected line as it crosses the perspective reference line. By plotting the different station values, the continuous roadway line maps the image as seen from the driver's eye.

Before a plot can be made, a relation between the driver, his visual cutoff point, and the horizontal curvature in the roadway must be established. Five cases will be considered as follows:

Case I, Figure 3b, is that for which both the driver and the maximum point of vision are positioned on the same tangent line, but neither lies in a horizontal curve. A-A represents the reference plane, and B, the maximum point of vision. Angle \( \alpha \) is the angle formed by the tangent line and the reference plane A-A. The distance X in feet is calculated by subtracting the position of the driver from the maximum point of vision. The distance along the reference line is

\[
\text{SUM X} = \cos |\alpha \| X \]

Case II, Figure 3c, is that for which neither the driver nor the maximum visual station lie in a curve, nor on the same tangent line. The distance X is calculated to the point of curve as in Case I. The tangent from the point of curve to C is derived as
TRANSLATION OF STATION POINTS

CASE I
3a DRIVER AND POINT ON TANGENT LINE

CASE II
3c DRIVER AND POINT NOT ON TANGENT LINE

CASE III
3d MAXIMUM VISUAL POINT IN CURVATURE

PERSPECTIVE CONCEPTS

FIGURE 3
\[ T = (R) \sin I \]  

where \( T \) is the tangent length, \( R \) is the radius of the horizontal curve, and \( I \) is the intersection angle of the two tangents. The reference plane distance may be calculated using equation (7) with the substitution of \( T \) for \( X \). The driver has effectively been advanced along the roadway from point of curvature to point of tangency. A comparison is needed to see whether the new driver position and the maximum point of vision now lie on the same tangent line. If not, the process is repeated, but if so, distance \( X \) becomes the difference between the maximum visual station and the driver location; equation (8) is again utilized. The various segmented reference plane distances are added.

Case III, Figure 3d, arises when the maximum visual station lies on the curve, but the driver does not. The driver is advanced according to Cases I and II and the respective reference plane A-A distances are evaluated until the point of curve is reached. The arc distance between station point B and the point of curve gives the distance along the curve to the maximum point of vision. The angle subtended by the arc is determined as

\[ A = \frac{(D) \cdot (X)}{100} \]  

where \( D \) is the degree of the horizontal curve and \( X \) is the distance along the arc.

Case IV and Case V are variations of the above mentioned cases in which either the driver or the maximum visual station are positioned along the lengths of horizontal curvature. If the driver's position lies in the curve but the visual point does not, Case IV is considered. Repeated applications of equations (7) and (8) are utilized to obtain the horizontal distance.

Next, the driver's vision line must be considered. All perspective distances are to be calculated as perpendicular distances from this sight line. By knowing the reference plane distance from the driver to his visual cutoff and the angle of the driver's sight line, we may calculate the distance from the driver to his visual cutoff according to

\[ \text{DIST} = \frac{R \text{ Dist}}{\cos \alpha_2} \]  

where \( \alpha_2 \) is the angle formed by the driver's sight line and the reference plane. \( R \) Dist is the reference plane distance from driver to vision cutoff. This scheme may also be used to determine the distances from the driver to any station point in his visual input. By a succession of angle
measurements, the angle formed by the reference plane and a station point projection can be calculated. By addition we now have the angle formed by the projection and the driver's vision line. Having chosen the perspective reference plane as a constant distance in front of the driver, the perspective horizontal value may now be obtained.

In similar manner, by replacing reference distances by the respective station point elevations the perspective vertical values may be calculated. This procedure may be repeated to include all the necessary projections.

ROADWAY SIGNING

The typical section of roadway contains many obstructions that are visible to the driver. In order to describe the roadway accurately it is necessary to represent these obstructions as the driver sees them. A limitation exists at present in the nature of the signing routine as the object must be represented by X and Y coordinate points. Curvilinear surfaces must be represented as successions of straight lines. As before, these points may be referenced to the roadway and their respective projections calculated to determine perspective values. This aspect becomes quite interesting since one is able to determine the visibility of signs under ideal conditions, using height, shape, and position as variables. Large objects such as bridge segments may be calculated in the same manner, using X and Y coordinates to describe their position in relation to the roadway.

HORIZONTAL SIGHT DISTANCE

The driver's horizontal field of vision is controlled to a great extent by the presence of objects in his visual path. For example, if a large sign is placed near the roadway, the driver can certainly be limited in his view of the roadway if any horizontal curvature is present. To accurately represent the driver's view of the roadway, his pictured visual input must be limited by these obstructions. For each obstruction minimum and maximum values must be formed, and the perspective values from each projection must be compared with these values.
The basic algorithms of this plotting scheme are especially adaptable to computer use since the numerous calculations necessary to align both the roadway and the driver can be performed accurately and rapidly by the computer. Also the large amount of design data needed to describe the roadway adequately can be readily stored. It also gives the engineer the flexibility to study many designs using the same basic algorithms and, if changes are necessary, the data may be quickly and easily altered to provide the changed parameters.

The algorithms are combined into one basic package consisting of a FORTRAN IV program that calls a number of FORTRAN IV subprograms as they are needed to accomplish the various steps. The fundamental principles of these routines have been presented earlier and this chapter describes computer implementation of these in the programs. A sample problem is reviewed and sample output of typical roadway sections is given in the appendix.

Initially, the basic roadway characteristics must be supplied to the computer with the test roadway stationed in some convenient increment. Design data are usually available in 100-foot station increments, but it was felt that these increments did not provide an accurate picture of the roadway for plotting purposes, so a simple interpolation scheme was utilized to provide the necessary information to the plotting program for 25-foot stations. A reference line must be established along the roadway, and in most cases, the center line of the roadway serves as an adequate reference line. Horizontal measurements to the roadway edges are entered as positive or negative perpendicular distances from the reference line. These distances must be supplied at each successive station point along the distance of the test roadway. The perpendicular distances to all continuous lines, including lane lines, medians, or curbs, may be entered for each station point if they are desired in the output drawing. The perpendicular distances are entered successively for each station point and each continuous line until all the horizontal distances are entered.
The main calling routine transfers control to the sub-program ELEVATE to determine the elevations of the various station points along the reference line. The plotting package assumes all gradient changes along the test roadway to be parabolic in shape and of the form

\[ Y = kX^2 \]  

(11)

with vertical distances as the Y coordinates, and the horizontal distances as the X coordinates. The rate of vertical curvature is entered for each vertical curve along the roadway and for each design speed. The tangent grades are entered in percent, plus for upgrades and minus for downgrades, for each successive vertical curve along the test roadway. The elevation of each station point along the reference line is then calculated using tangent grades as they are needed. The general procedure of the elevation calculations is outlined in Figure 4.

Using equation (3) for vertical curvature, the elevations of all the station points along the reference line are stored for later use. Also computed is the angle \( \theta \) between the driver's eye and the horizontal plane at each station point. The \( \theta \) angle is recorded as positive for angles above the horizontal and as minus for angles below the horizontal. The data are needed later in the program to determine maximum vision point from the driver's eye.

After completion of the elevation calculations of station points along the reference line, sub-program HEIGHT is called to calculate the elevations of all points along the continuous lines of the roadway. Variable ASLOPE is the value for the side slope of the roadway perpendicular to the reference line and must be entered in the form feet-per-twelve feet. Utilizing the perpendicular distances to the continuous roadway lines, the station point elevations are calculated successively and control is again transferred to sub-program ELEVATE. Output from the elevation sub-programs includes vertical points of curvature and tangency for each vertical curve, the tangent grades in percent for each vertical curve, the perpendicular distances to each continuous line at each station point, the elevation in feet of all station points on each continuous line, and the angle between the driver's eye and the horizontal plane at each station point.

After all elevations are calculated on the test roadway, the plotting program now must orient the roadway horizontally. An imaginary reference plane is designated to lie adjacent to the roadway and sub-program CURVE is used to find the horizontal distance from the reference line to the reference plane at all station points along the roadway.
CALCULATE LENGTH OF VERTICAL CURVE

IS CURVE PRECEDED BY CONSTANT GRADE

CALCULATE OFFSETS FROM CURVE TO STATIONS

CALCULATE ELEVATIONS FOR STATION POINTS

CALCULATE DRIVER'S EYE ANGLE

ARE THERE MORE CURVES

CALCULATE RATE OF ELEVATION CHANGE

COMPUTE ELEVATION OF STATIONS TO CURVE

RETURN

FIGURE 4
ELEVATION SUBPROGRAM
By simply entering the points of intersection of all the horizontal curves and the corresponding tangent angles as related to the direction of the reference plane, the horizontal distance to the reference line at all station points may be calculated. The point of intersection in stations and the radius of curvature in feet are needed. The tangent angles as related to the reference plane are entered in radians. For example, if the tangent moves in a direction to the right of the reference plane, it is entered as a positive quantity and if the tangent moves in a direction to the left of the reference plane, it is entered as a negative quantity. An indicator is needed to designate the concave side of the horizontal curve.

The tangent length is computed to determine the point of curve. If the point of curve lies beyond the point of tangency of the previous horizontal curve, the line of constant slope yields the reference distance according to

\[
\text{DIST}(I) = \text{DIST}(I-1) \pm 25 \cdot \sin \theta \quad (12)
\]

where DIST(I) is the perpendicular distance to the reference plane and \( \theta \) is the tangent angle. The minus sign holds if the tangent angle is negative in direction. After the point of curve is reached, the reference angle, formed initially by the radius vector and the reference plane, is computed. By successive additions to the reference angle, as seen in Figure 2b, the reference distance may be calculated. These computed reference distances are stored in memory at each station point. For 25-foot incremental stationing as previously chosen, the angle to be added to the reference angle for each station point corresponds to the degree of the curve divided by four. This algorithm will allow angles to a maximum of 180 degrees. Figure 5 illustrates the sequence of steps in determining reference plane distances.

An optional feature of the plotting package allows the designer to obtain the \( \alpha_2 \) angle of Figure 2b formed by the driver and the reference plane, if it is necessary. The angle is obtained by successive additions or subtractions from the original \( \alpha_2 \) angle. Then it is saved for later use in the perspective calculations. By entering horizontal curve data as they are needed and making the subsequent calculations, the reference line becomes oriented horizontally to the reference plane. Sub-program SDIST is now utilized to adjust the station points on each continuous roadway line by adding their respective perpendicular distances from the reference plane to the oriented reference line.
INPUT HORIZONTAL CURVE DATA

CALCULATE TANGENT LENGTH AND POINT OF CURVE

IS CURVE PRECEDED BY CONSTANT SLOPE

YES

CALCULATE RATE OF CHANGE USING TANGENT

NO

DETERMINE REFERENCE ANGLE

INCREMENT REFERENCE ANGLE

COMPUTE DISTANCE TO REFERENCE PLANE

IS END OF CURVE REACHED

NO

YES

ARE THERE MORE CURVES

YES

RETURN

NO

FIGURE 5
HORIZONTAL CURVATURE
SIGHT DISTANCE CALCULATION

The roadway is now aligned in a Cartesian coordinate system and we are ready to consider the driver on the roadway. The driver is positioned by a data entry giving his location in station points. The first problem encountered is the sight distance of the driver. The plotting routine transfers control to sub-program CUTPT to determine the sight distance on vertical curves. Initially, the program has entered a maximum value of sight distance that may be shortened by the presence of limiting objects on the roadway. By a systematic comparison of angles between the driver's eye and roadway points in front of him, the maximum point of vision is calculated. Four cases are tested in this algorithm.

Case I, Figure 6a, is considered when the angle between the driver's eye and the horizontal is negative. This angle is the stored angle $\theta$ from the elevation calculation described earlier. DISTY, the change in elevation between the driver and the station point, is calculated by

$$\text{DISTY} = \text{EYEH} + \text{ELEV} - \text{ELEVP} \quad (13)$$

where EYEH is the height of the driver's eye above the roadway surface, DISTX is the difference in feet between the point in question and the driver location; ELEV is the elevation of the driver; and ELEVP is the elevation of the point in question. Consequently, the angle $\beta$ is calculated for each station point as

$$\beta = \arctan \left( \frac{\text{DISTY}}{\text{DISTX}} \right) \quad (14)$$

where DISTY is the change in elevation between the driver and the station point. Successive station points are tested until the angle $\beta$ becomes larger in magnitude than the preceding one. This indicates that the point in question is not visible to the driver and the previous station point is designated the maximum sight distance for the driver.

Case II, Figure 6b, is also considered when the angle between the driver's eye and the horizontal is negative. DISTY, DISTX, and the beta angle are calculated as before using equations (13) and (14). In this case the driver's sight distance is not impeded by the roadway surface, but headlight sight distance becomes somewhat of a controlling factor. General practice allows a 1-degree upward divergence of the light beam from the longitudinal axis of the vehicle (1). If the angle $\beta$ becomes more than 1-degree less than the angle $\theta$ for the present driver location, a visual maximum has been reached and the station point becomes the maximum sight distance for the driver.
When the angle between the driver's eye and the horizontal is positive, the program utilized Case III (Figure 6c). DISTY is now calculated as

\[
\text{DISTY} = \text{ELEVP} - \text{ELEVD} - \text{EYEH} \quad (15)
\]

with ELEVP and ELEVD defined as in equation (13). Again the headlight sight distance becomes a factor, and the Case II method is repeated to yield the maximum distance.

Finally, Case IV, Figure 6d, is considered when the angle is positive with DIXTX and DISTY derived as in Case III. The successive \(\beta\) angles are compared as in Case I until the driver's vision is hindered by the roadway surface. Again, this station point is stored as a maximum sight distance.

In all the above mentioned cases, the difference between the station point in question and the driver location is compared to the maximum allowable sight distance. If this maximum sight distance is reached, further \(\beta\) angle comparisons are terminated.

**PERSPECTIVE SUBPROGRAMS**

To arrive at the perspective picture of the roadway, the program transfers control to sub-program XPERSP. Since the test roadway may contain sections of horizontal curvature, all measurements of distance must be first translated to the reference plane established earlier in the program. The location of the driver is checked by sub-program PLACE as to his position relative to any horizontal curvature. The station point of the vehicle is compared to the point of curve and point of tangent of any horizontal curves calculated earlier in the program, and appropriate indicators are set. An identical scheme is utilized to indicate the position of the maximum visual point.

Sub-program DISCAL is now employed to calculate the distance along the reference plane from the driver to the maximum visual point. Five options are handled in DISCAL according to the positions of the driver and maximum sight distance as indicated by PLACE with indicators, set by PLACE, determining which option is needed to calculate the reference plane measurement. All horizontal curve data have been saved from earlier calculations and are available to sub-program DISCAL.
CASE I
6a SIGHT DISTANCE ON NEGATIVE GRADE

CASE II
6b SIGHT DISTANCE ON NEGATIVE GRADE

CASE III
6c SIGHT DISTANCE ON POSITIVE GRADE

CASE IV
6d SIGHT DISTANCE ON POSITIVE GRADE

SIGHT DISTANCES CONCEPTS

FIGURE 6
The designer is allowed one of two choices in determining the vision line of the driver: (a) the driver's vision line is directed at the maximum visual point, (b) the driver's vision line is directed along the path of this vehicle. Let us consider the first choice. In Figure 7a, distance X is calculated by taking the difference in the perpendicular distances from the reference plane to the station points. SUMX is the previously derived distance along the reference plane from the driver to the maximum visual point. \( \alpha_1 \), termed the reference angle, is determined by

\[
\alpha_1 = 90^\circ - \arctan \left( \frac{\text{SUMX}}{X} \right)
\]

and this angle describes the relation between the vision line from the driver's eye and a parallel to the reference plane. The slope of the vision line is also calculated at this point and saved for later comparisons. The distance along the driver's vision line from the driver to the maximum visual point is obtained by use of the Pythagorean Theorem.

Now considering the choice (b) of the driver's vision line along the path of the vehicle (Figure 7b) in which case the reference angle, \( \alpha_1 \), is a previously defined quantity. The distance along the driver's vision line is calculated using the reference plane distance SUMX. The perpendicular difference between the two station points is calculated as

\[
X = (\text{XDIST}) \sin |\alpha_1|
\]

where XDIST is the distance along the vision line. The perpendicular distance from the reference plane to the maximum visual point C in Figure 7b is derived as the sum of the perpendicular distance to the vehicle plus the distance X.

The program now begins a successive translation of the points along each continuous line between the perspective reference line and the maximum visual point. Each point in turn is positioned relative to any existing horizontal curvature and its distance along the reference plane from the driver is calculated using sub-program DISCAL. The perpendicular distance from the driver to the point in question is determined from the reference plane. The slope of the line from the driver to the station point is now calculated and the related angle formed by the line and the reference plane is arrived at using equation (14).

A comparison is made to determine the relation between the point in question and the maximum visual point. If they lie on different sides of the driver as in Figure 7c, the two relative angles add to give the total angle formed by the driver's vision line and the line from the driver to the point. Since the perspective reference line B-B is perpendicular and taken to be fifty feet in front of the driver, the perspective distance X for the point is calculated as

\[
X = \tan \alpha_1
\]
and if the slope of the line from the driver to the station point is less than the slope of the driver's vision line, the perspective distance $X$ becomes negative indicating the point lies to the left of the driver. If the relation between the station point and the maximum visual point is such that they lie on the same side of the driver, their respective reference angles are subtracted and calculations are made as before.

Control is now transferred to sub-program YPERSP to calculate the perspective elevation of the selected station point. From the angular relations derived about the station point it becomes a simple matter to calculate the distance along the line from the driver to the point. The distance from the driver to the perspective reference line, $DIST$, is given as

$$\text{DIST} = \frac{50}{\cos |\alpha|}$$  \hspace{1cm} (19)

where $\alpha$ is the total angle formed between the line from the driver to the station point and the driver's vision line. From these two distances a relation is formed to yield the perspective elevation.

Figure 7d illustrates the case where the elevation of the object B lies below the elevation of the driver's eye, $DIST$ represents the distance from the driver to the point. The relation

$$Y = \frac{(\text{EYE HEIGHT})(X - \text{DIST})}{X}$$  \hspace{1cm} (20)

yields the perspective Y value in feet. This value is added to the elevation of the station point B to produce the desired perspective elevation for the station point. If the elevation of the station point is higher than the elevation of the driver's eye the perspective Y value must be added to the elevation of the driver's eye to yield the perspective elevation.

The equations mentioned above are used repeatedly until a perspective X distance and a perspective Y elevation are calculated for each station point along the continuous lines of the roadway within the limits of the driver's vision. Figure 8 illustrates the general outline of the perspective calculations.

**OBSTRUCTION PLOTTING**

Methods tested in this plotting program to represent obstructions along the roadway are very similar to those described earlier. The in-
7a SIGHT DIRECTION AT MAXIMUM VISUAL POINT

7b SIGHT DIRECTION ALONG PATH OF VEHICLE

7c ANGULAR RELATIONS

7d POINT LYING BELOW EYE LEVEL

SIGHT DISTANCE CONCEPTS

FIGURE 7
READ DRIVER LOCATION

REFERENCE PLANE DISTANCE TO CUTOFF

READ MAXIMUM VISION

CALCULATE POSITIONS TO HORIZONTAL CURVES

CALCULATE X PERSPECTIVE VALUE

CALCULATE Y PERSPECTIVE VALUE

ARE ALL STATIONS CALCULATED

INCREMENT LINE NUMBER

INCREMENT STATION NUMBER

ARE ALL LINES CALCULATED

RETURN

FIGURE 8
PERSPECTIVE MODEL
Individual objects are entered as data with the appropriate station point and the number of data points as header words. For example, the typical roadway sign of Figure 9a would contain seven data points. For each data point X and Y values must be supplied with the X value being distance between the point and the reference line of the roadway, and the Y value being the height of the point above the roadway. It is obvious that almost any roadway object may be broken into X and Y points and thus may be represented in this manner. Sub-program SDIST is called to align the object with the reference plane previously established in the program. This is simply done by addition or subtraction to the existing roadway points.

After the driver’s position has been established and the basic roadway points have been drawn in perspective, sub-program SVALUE is called by the plotting program to determine perspective values for the roadway objects. SVALUE compares the station position of each object with the driver location and his maximum visual point to determine whether the object lies within the visual field of the driver. If the object is visible to the driver a similar scheme described earlier is used to determine the perspective calculations of each data point of the object. The perspective points are stored in a like manner to the original data points with the two header words being object position in stations and number of data points. Each object is checked in turn and control is transferred to the main calling program.

PLOTTING PROGRAM

The plotting programs are governed by a parameter card allowing plotting of a profile, plan, or perspective view of the roadway, or any combination of these. No attempt is made to plot the roadway obstructions in plan or profile views. The plotting program generates an output tape of pen movements sufficient to plot the desired view on the Model 565 digital plotter connected to an IBM 1401 system.

If a plan view is desired the aligned roadway values at each station point are plotted versus the distance of the roadway. Continuous lines are drawn through the station points along the length of the roadway. If a profile view is desired, the elevation of the reference line is plotted versus the distance of the roadway. Again, a continuous line is drawn through the station points along the length of the roadway.

If a perspective plot is desired the scheme becomes more involved because of the limit in sight distance due to the presence of objects on
the roadway. Sub-program CALPLT is used to plot the perspective values. Initially, the roadway objects that lie within the driver's visual field must be plotted. A maximum and minimum X value and a maximum and minimum Y value are determined for each object. The nearest object to the driver's eye that lies within his visual field is plotted with a continuous line connecting the X and Y values to represent the object in perspective. A comparison of coordinates of the next object is made to see whether they lie within an area not visible to the driver because of the presence of the nearer object. If the points are visible, they are plotted and a continuous line is drawn connecting all visible points. This comparison is repeated until the visible points of all the objects within the driver's visual field have been plotted. For example, in Figure 9b, point 3 is not visible to the driver from his particular location. Sign A would be plotted but only points 1, 2, 4, 5, and 6 of sign B would be plotted.

When all the objects have been plotted the basic roadway may be drawn. Each continuous line along the roadway is entered in turn and the perspective X and Y values of each station point are compared with the minimum and maximum values of the objects to determine whether the station point of the roadway is visible to the driver. If the points are visible, they are plotted and a continuous line is drawn through them. If the point is not visible to the driver, the continuous line is broken until further comparisons indicate a visible point.

Because of the presence of object A of Figure 9C, station point 6 of the roadway line is not visible to the driver, and the plotted line becomes discontinuous at station point 5. Comparison indicates that station point 7 is visible to the driver, and the line continues from this point. Plotting is continued until all the visible station points of the roadway lines within the driver's visual field are drawn.

SUMMARY OF RESULTS

This study concerned the development of the necessary plotting algorithms to graphically represent the configuration of a roadway as seen from the driver's eye using design information as data. These algorithms were developed, evaluated and their applications to the problem studied. The following summary of results was prepared as a result of this research:
VISUAL LIMITATIONS

SIGN PLOTTING CONCEPTS

FIGURE 9
1. The methods described provide a useful new tool to be used in the design of roadways to fit the driver's needs. Any object along the roadway that may be represented in X and Y coordinate values may now be viewed in perspective.

2. Sections of roadway can be segmented into 25-foot station lengths and a typical driver advanced along its length of yielding graphical displays of his visual input at each station.

3. With data collected from typical roadways around College Station, Texas, perspective pictures have been plotted using the algorithms developed in this study. Sample graphs utilizing data from existing roadways are included in the Appendix of this report.

4. In instances where such were available the computed results were checked with existing data and a reasonable accuracy of representation was evident in all cases tested.

RECOMMENDATIONS

The following recommendations for research are made:

1. Additional refinement of the concepts and methodology of the plotting routines is needed to make the procedure applicable to the evaluation of roadway design.

2. For "production" work, the programs should be further modified in order to minimize storage requirements and running time. At present, approximately 3 minutes of IBM 7094 computer time and 4 minutes of Digital Plotter time are needed to calculate and draw 20 perspective views of a typical roadway.

3. Further refinement of the algorithms is needed to accurately represent bridge layouts or multilane, multilevel interchanges.

4. It also seems feasible to utilize the methodology to graphically describe objects not related to the transportation field. The visual inputs to an observer of an object not having curvilinear sections may be represented. For example, the field of architectural design might utilize the program to view proposed architectural constructions in perspective. Consideration is being given to the use of the program to draw certain anatomical features of dissected biological specimens in a similar manner.
REFERENCES


