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16. Abstract

As is the case for most of the Departments of Transportation in the U.S., the Texas Department of Transportation has been experiencing fluctuations of budget for maintaining and preserving its highway infrastructure over the recent years. If the maintenance budget shortfall lasts for an extended period of time, the condition of the highway network would be harmed directly or indirectly since some maintenance work would be deferred or cancelled. Thus, in order to control and minimize the risk caused by maintenance budget reductions, it is important for highway agencies to adjust their maintenance and rehabilitation policies to accommodate budget fluctuations. This report presents a methodological framework that helps highway agencies quantify the risks to highway networks, and revise the highway routine maintenance work plans to minimize the impact of budget fluctuations. The proposed methodology aims to assist highway agencies in prioritizing and selecting maintenance functions according to the risk of not performing a specific maintenance activity. Also, this methodology considers the subjective nature of decision makers' assessments, allowing different levels of confidence and different attitudes toward risk to be captured as the uncertainty and imprecision involved in the decision making process. In the case study, the proposed methodology is tested with a set of data obtained from the Texas Department of Transportation. The result is compared with the outcome obtained from the crisp Analytical Hierarchy Process using the same set of data. The outcomes from the two methodologies are very close, validating the effectiveness of prioritizing highway maintenance functions using Multi-Attribute Analysis with Fuzzy Pairwise Comparison.

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PRIORITIZATION OF HIGHWAY MAINTENANCE FUNCTIONS USING MULTI-ATTRIBUTE DECISION MAKING WITH FUZZY PAIRWISE COMPARISON

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EXECUTIVE SUMMARY

Multi-Attribute Analysis with Fuzzy Pairwise Comparison is a methodology aimed at supporting decision makers to deal with multiple and sometimes conflicting attributes in prioritization. Pairwise comparisons of routine highway maintenance functions are conducted with experts and engineers working in highway maintenance related fields. Due to the subjective nature of human judgments, the assessments are processed as fuzzy numbers. In this study, decision makers are asked to assess the relative importance of one maintenance function over another, with respect to different maintenance objectives. Since these assessments are usually derived or interpreted subjectively, uncertainty and imprecision are involved. In order to capture the subjectivity and imprecision in the assessments, fuzzy logic is incorporated to handle the subjectivity of the assessments.

There are five main components in this methodology: 1) obtain the decision matrix of maintenance functions by fuzzy synthetic extent analysis; 2) obtain the fuzzy performances of all maintenance functions; 3) incorporate decision maker's degree of confidence into fuzzy performances; 4) incorporate decision maker's attitude toward risk into the maintenance function performances; 5) build a ranking index based on maintenance function performances, and rank the highway maintenance functions.

As is the case for most of the Departments of Transportation in the U.S., the Texas Department of Transportation has been experiencing fluctuations of budget for maintaining and preserving its highway infrastructure over the recent years. If the maintenance budget shortfall lasts for an extended period of time, the condition of the highway network would be harmed directly or indirectly since some maintenance work would be deferred or cancelled. Thus, in order to control and minimize the risk caused by maintenance budget reductions, it is important for highway agencies to adjust their maintenance and rehabilitation policies to accommodate budget fluctuations. This report presents a methodological framework that helps highway agencies quantify the risks to highway networks and revise the highway routine maintenance work plans to minimize the impact of budget fluctuations. The proposed methodology aims to assist highway agencies in prioritizing and selecting maintenance functions according to the risk of not performing a specific maintenance activity. Also, this methodology considers the subjective nature of decision makers' assessments, allowing different levels of confidence and different attitudes toward risk to be captured as the uncertainty and imprecision involved in the decision making process. In the case study, the proposed methodology is tested with a set of data obtained from the Texas Department of Transportation. The result is compared with the outcome obtained from the crisp Analytical Hierarchy Process using the same set of data. The outcomes from the two methodologies are very close, validating the effectiveness of prioritizing highway maintenance functions using Multi-Attribute Analysis with Fuzzy Pairwise Comparison.

DISCLAIMER

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CHAPTER 1. INTRODUCTION AND BACKGROUND

The Texas Department of Transportation (TxDOT) has been suffering from a reduction in budget for maintaining and preserving the highway infrastructure. The reduction in the maintenance budget could lead to large scale deferred maintenance activities, both pavement related and non-pavement related maintenance functions. Deferring maintenance will have a negative impact on the highway network condition in the long run. Thus it is important for highway agencies to evaluate the risks and revise the work plans as needed to minimize the negative effect.

1.1 The Need for Prioritizing Maintenance Projects

As the maintenance budget shortage lasts for an extended period of time, some maintenance work would have to be deferred or cancelled. The condition of highway network would be negatively affected, since each deferred maintenance activity would cause a penalty in terms of the risk exposed to road users. As a result, there is an urgent need for controlling and minimizing the total risk caused by maintenance budget reduction. Therefore, highway agencies need to select and perform the most urgent projects within the limited budget, and revise their maintenance and rehabilitation work plans to accommodate budget fluctuations.

In order to prioritize and select maintenance activities under budget fluctuations, a feasible approach is needed to evaluate the risk of not performing a maintenance activity. This report proposes the use of Multi-Attribute Analysis with Fuzzy Pairwise Comparison to quantify those risks. The proposed methodology evaluates the risks of each maintenance activity from multiple aspects, and then uses the quantified risks to rank the maintenance projects. Based on the risk quantification, highway agencies are able to prioritize the maintenance functions, and revise the work plans to minimize the risk due to budget reduction.

1.2 The Importance of Routine Highway Maintenance

The concept of routine highway maintenance extends pavement management to various maintenance categories. Besides pavement related maintenance, other types of non-pavement related maintenance, such as traffic operations related maintenance, roadside related maintenance, and bridge related maintenance are also included in routine highway maintenance.

In the past, highway agencies allocated budget and resources to pavement related maintenance functions with the highest priority. But more and more highway agencies now realize that, other types of maintenance functions are as important as pavement related maintenance functions. In some cases, deferring a non-pavement related function would be more detrimental to highway networks in the long run.

Take grass mowing as an example, grass mowing is often compromised when there is a shortage in the budget. People would consider grass mowing to be less important, since grass mowing will not immediately impact the ride quality. Thus people tend to underestimate its impact. However, ride quality is impacted by grass mowing indirectly. If the grass were not mowed for a period of time, animals would hide inside the grass potentially resulting in hazardous crashes. Grass mowing is important not only from the safety aspect, it is also important from the pavement preservation aspect. If the grass were not mowed regularly, grass roots could get into the pavement structure and weaken the pavement strength. [1]

Therefore, it is not wise to put all the efforts and limited budget into pavement related maintenance. Non-pavement related maintenance functions should also be taken seriously.

Highway agencies should recognize the importance of routine highway maintenance and pay more attention to it.

1.3 Routine Highway Maintenance Prioritization in Texas

There is a growing need for routine highway maintenance prioritization in almost every state. Many of the state DOTs, including TxDOT, have adopted asset management systems to help evaluate and determine the optimal timing for key maintenance activities. However, TxDOT has gone through budget fluctuations in highway infrastructure maintenance and preservation over the last few years. As a result of budget reduction, some of the scheduled maintenance activities cannot be performed on time. The budget fluctuations can potentially harm the highway condition, since some maintenance activities are deferred or cancelled for the time being. Thus, TxDOT needs to establish maintenance and rehabilitation strategies to minimize the risk caused by the reduction in highway network conditions.

CHAPTER 2. LITERATURE REVIEW

A literature review was conducted to investigate existing methods and techniques for quantifying and assessing project risk. Although there are many different risk assessment methods available, the fundamental elements of risk assessment processes are identical: identifying hazards, assessing risk, reducing risk, and documenting the results. [2]

Bruce W. Main summarizes the typical risk assessment process in 7 Steps, as shown in Figure 1. [2] The following discussion will provide a sketch of assessing the risks of routine highway maintenance functions.

Step 1: Set the limits/scope of the analysis;

First of all, the goal of risk assessment needs to be clarified. In this study, the goal is quantifying the risks and impacts of deferring or cancelling routine highway maintenance due to budget cuts. Also, the scope of the analysis is to be defined prior to any analysis.

Step 2: Identify tasks and hazards;

For any project risk assessment problem, the tasks need to be clearly defined, and the number of tasks needs to be controlled. For highway routine maintenance prioritization, the number of maintenance functions is to be determined to prevent the problem from growing to an unmanageable size. And the key routine highway maintenance functions are to be selected.

Moreover, the risk of deferring a maintenance function should be evaluated from multiple aspects. For example, not mowing the grass could cause safety issues, system preservation and aesthetics problems. Therefore in this case, safety, system preservation, and aesthetics are the maintenance objectives. For the maintenance function risk assessment problem, the maintenance objectives should be carefully selected.

Step 3: Assess risk;

As maintenance functions and maintenance objectives are defined, the risk of each maintenance function is to be assessed and quantified with respect to every maintenance objective. The overall risk of a specific maintenance function is obtained by integrating the risks associated with different maintenance objectives. There are various risk scoring approaches available, such as Simple Additive Scoring System, Risk Priority Numbers, Analytical Hierarchy Process, etc. In this report, Multi-Attribute Analysis with Fuzzy Pairwise Comparison is adopted to quantify the risks of routine highway maintenance functions.

Step 4: Minimize risk;

When there is a budget cut, some maintenance functions will be deferred or cancelled. The risks of deferring these maintenance functions can be quantified through Step 1 to Step 3. The maintenance functions can therefore be ranked according to descending order of the risks. Select the maintenance functions from the top of the list to the bottom, until the budget is exhausted. In this way, the total risk induced by budget cut is minimized.

Step 5: Assess risk– Residual;

In some occasions, there is a maximum tolerance level for the total risk. And the summation of risks of all deferred maintenance activities should be less than the acceptable risk. If not, go back to Step 2.

Step 6: Decision making process;

If the total risk of selected maintenance functions in Step 4 is acceptable, perform those maintenance functions. The unselected maintenance functions are deferred or cancelled for the time being.

Step 7: Output results/documents.

Output the selected maintenance functions, and the corresponding risks. Reschedule the maintenance activities according to the outputs.

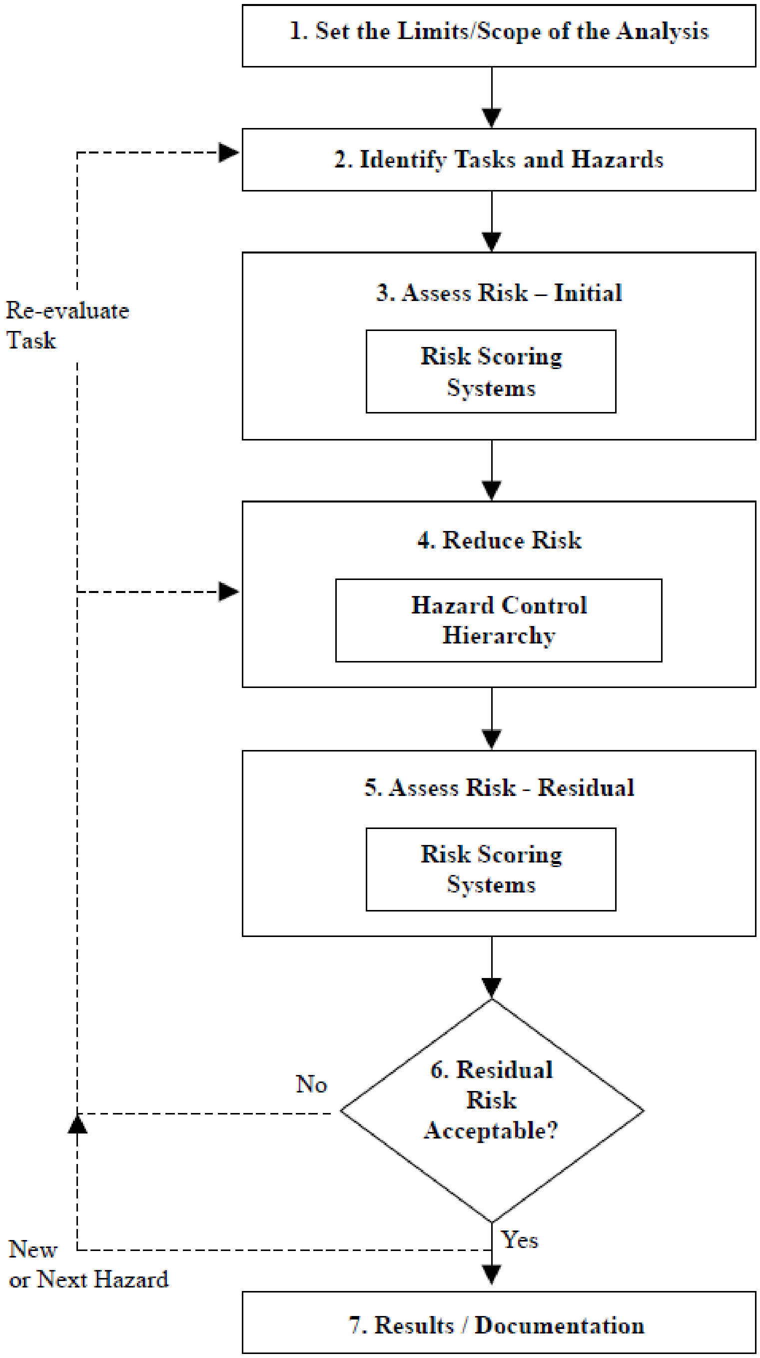


Figure 1. The Risk Assessment Process.

2.1 PROBABILISTIC APPROACH TO QUANTIFYING RISK

Risk can be assessed based on reliability analysis. Qiang Meng et al. proposed a probabilistic quantitative risk assessment model, which consisted of a work zone crash frequency estimation, an event tree, and consequence estimation models [3]. There are seven intermediate events – age (A), crash unit (CU), vehicle type (VT), alcohol (AL), light condition (LC), crash type (CT) and severity (S) – in the event tree.

The estimated values of probability for some intermediate events may have high uncertainty. And the uncertainty can be characterized by random variables. The consequence estimation model takes into account the combination effects of speed and emergency medical service response time (ERT) on the consequences of work zone crashes.

Probabilistic approaches are able to give crisp-value assessments. But due to the subjectivity of risk evaluations, the effectiveness of these assessments is to be questioned.

2.2 RISK PRIORITY NUMBERS

Zaifang Zhang et al. used Risk Priority Numbers to quantify the risks of different failure modes for a drilling machine [4]. In the Risk Priority Numbers approach, the risk of each alternative is assessed from three aspects: 1) severity, which rates the severity of the potential effect of the failure; 2) occurrence, which rates the likelihood that the failure will occur; 3) detection, which rates the likelihood that the problem will be detected before it reaches the end. And the total risk is obtained by multiplying the likelihood ratings of severity, occurrence and detection.

The simplicity of RPNs makes it popular in practical use, but the disadvantage of this approach is also obvious: the decisions are based solely on severity, occurrence and detection, which may result in inefficiency and increased risk.

2.3ANALYTIC HIERARCHY PROCESS

Analytical Hierarchy Process (AHP) has been successfully used in different fields and disciplines. Its ability to handle both qualitative as well as quantitative data makes AHP an ideal methodology for some prioritization problems. There has been extensive research on prioritizing using the AHP method. The fundamental logic of AHP is to decompose a large complex task into smaller and manageable subtasks. In essence, AHP enables users to create different levels or hierarchies depending on the complexity of the problem. Furthermore, the projects are prioritized based on pairwise comparison assessments. Each pairwise comparison assessment is obtained by comparing two alternatives at a time, and a relative value is assigned to each pair. Using AHP, a priority vector of the alternatives is developed from the synthesis of the pairwise comparisons.

One of the AHP applications is the selection of petroleum pipeline routes. The petroleum pipeline route is the connection of the crude/natural gas source to the refinery or utility company. Obviously, choosing the shortest, most direct route is always a goal for capital expenditure reasons. However, sometimes geophysical, environmental, political, social, economic, and regulatory factors may conflict with choosing the shortest pipeline route, and makes the shortest route a bad choice. Sam Nataraj applies AHP to evaluate the risks of possible routes with respect to each attribute. The total risk for each route is obtained based on pairwise comparisons. And the pipeline route with minimum risk is therefore the best choice [5].

In Electric Power Systems Research, there are different demand response (DR) programs for improving load profile characteristics and achieving customer satisfaction. H. A. Aalami et al. used an economic model, MAMD techniques, including entropy, TOPSIS, and AHP to help power market regulator set rules for selecting and prioritizing DR programs in the power market [6]. Aalami's study shows AHP can be used to deal with multiple market operation problems such as price spikes, insufficient spinning reserve margin, system security and reliability.

Tsuen-Ho Hsu et al. report in their study the development of a comprehensive model that measures dental service quality using AHP [7]. AHP is used to examine the quality structure of dental services. Since pairwise comparisons could be viewed as random variables with certain distributions, Monte Carlo simulation is integrated into the model. Results from this model provide strong guidance for the management of dental clinics, and have significant cost-saving and revenue-increasing contributions. Their study extends the applications of both AHP and the Monte Carlo simulation in service industry management, and proves the Monte Carlo-AHP approach's ability in prioritizing critical attributes. Also, this Monte Carlo-AHP approach greatly sharpens the effectiveness of the decision-making process.

CHAPTER 3. METHODOLOGICAL FRAMEWORK

In this chapter, the methodology of prioritizing routine highway maintenance functions using Multi-Attribute Analysis with Fuzzy Pairwise Comparison is introduced. Multi-Attribute Analysis with Fuzzy Pairwise Comparison is a methodology aimed at supporting decision makers to deal with multiple and sometimes conflicting attributes in prioritization. In this study, decision makers are asked to assess the relative importance of one maintenance function over another, with respect to different maintenance objectives. Since these assessments are usually derived or interpreted subjectively, uncertainty and imprecision are involved. In order to capture the subjectivity and imprecision in the assessments, fuzzy logic is incorporated to handle the subjectivity of the assessments.

3.1 BASIC CONCEPTS OF FUZZY PAIRWISE COMPARISON

Pairwise comparisons of routine highway maintenance functions are conducted with experts and engineers working in highway maintenance related fields. Due to the subjective nature of human judgments, the assessments are processed as fuzzy numbers. To understand the proposed approach, it is necessary to comprehend the concepts associated with pairwise comparison and fuzzy logic. Thus before moving on to the conceptual framework, basic concepts of pairwise comparison and fuzzy logic are briefly discussed as follows.

3.1.1 Pairwise Comparison Concept

Pairwise comparison is widely adopted in practice because of its simplicity. Pairwise comparison is a process that compares alternatives in pairs to determine which alternative is preferred. This approach is popular in acquiring subjective judgments, since it is much easier for the human brain to compare two items at one time, than to assign scores or weights to the alternatives when the number of alternatives exceeds three.

After the maintenance functions and maintenance objectives are defined, decision makers are asked to give pairwise comparisons on the maintenance functions under each maintenance objective. The evaluations are obtained based on the definitions and explanations from a Scale of Relative Importance Table, as shown in Table 1 [8]. Each comparison is evaluated under a specific objective, e.g., from the safety aspect, Maintenance Function 1 is weakly important over Maintenance Function 2, thus the pairwise comparison value is 3.

Table 1. Scale of Relative Importance.

Intensity of Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective.
3	Weak importance of one over the other	Experience and judgment slightly favor one over the other.
5	Essential or strong importance	An activity is strongly favored and its dominance demonstrated in practice.
7	Demonstrated importance	The evidence favoring over another is of highest possible order of affirmation.
9	Absolute importance	When compromise is needed.
2,4,6,8	Intermediate values	

3.1.2 Linguistic terms and fuzzy set theory

There are multiple objectives for performing maintenance functions, e.g., safety, ride quality, and aesthetics. In this study, most of the maintenance objectives are subjective terms, and it is difficult to obtain crisp and exact assessments. Moreover, a crisp value can hardly represent the uncertainty and imprecision involved in decision maker's judgments. Thus, fuzzy logic is used in this study to handle and process subjective pairwise comparisons.

3.1.2.1 Fuzzy Logic and Membership Function

In this report, all pairwise comparisons between maintenance functions are handled and processed as fuzzy numbers. Fuzzy logic is a form of multi-value logic derived from fuzzy set theory, which handles subjective or approximate reasoning, rather than objective and exact measurement. Different from traditional "crisp logic," the truth value of each fuzzy logic variable transits from 0 to 1, whereas the truth value of variables in crisp logic is either 0 or 1. [9]

In 1965, Zadeh first used fuzzy set theory to address problems involving fuzzy phenomena [10]. In a universe of discourse, membership functions on X represent fuzzy subsets. A fuzzy subset A of X is defined with a membership function denoted by $\mu_A(x)$. The membership function maps each element x in X to a real number in the interval [0,1].

Fuzzy subset A can be described as an objective in the form of $A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\}$, where $\mu_A(x)$ indicates the degree of membership, and $\gamma_A(x)$ indicates the degree of non-membership for element $x \in X$. And any element $x \in X$ satisfies $0 \le \mu_A(x) + \gamma_A(x) \le 1$ [10]. Membership functions may have different mathematical expressions. The simplest and most commonly used membership function is triangular fuzzy number function, as shown in Figure 2.

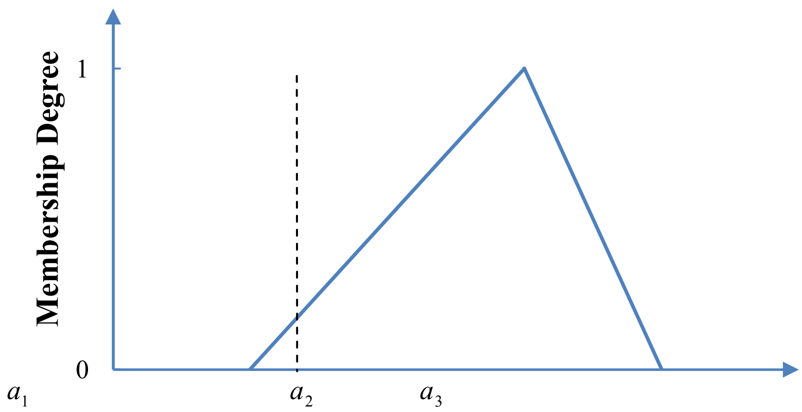


Figure 2. Triangular Fuzzy Number Function.

A triangular fuzzy number is usually denoted as (a_1, a_2, a_3), where a_2 is the center value, a_1 is the left displacement and a_3 is the right displacement. The membership function of a triangular fuzzy number is, [9]

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

where, a_1, a_2, a_3 are crisp numbers.

Fuzzy logic is widely used in dealing with subjective and approximate assessments. In this study, decision makers' assessments on pairwise comparisons of routine highway maintenance functions are processed as triangular fuzzy numbers.

3.1.2.2 Fuzzy Number Operations

The operations on fuzzy numbers are different from crisp numbers. There are three types of fuzzy number operations involved in this study: inverse, addition, and division. These three operations are defined as following.

Assume there are two fuzzy sets *A* and *B*, with positive fuzzy numbers (a_1, a_2, a_3) and (b_1, b_2, b_3), respectively. The basic fuzzy arithmetic operations on these fuzzy numbers are defined as, [9]

1) Inverse:

$$A^{-1} = (\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}).$$

2) Addition:

$$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3).$$

3) Division:

$$\frac{A}{B} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$$

3.1.2.3 α-cut Concept

α-cut of a fuzzy set is the crisp set of all elements that have a membership value greater than or equal to α [9]. For a fuzzy set A, its α-cut is described as $A_\alpha = \{x \in X / \mu_A(x) \geq \alpha, \gamma_A(x) \geq 0\}$, as shown in Figure 3. Subset A after α-cut can be denoted as $A_\alpha = [x_l^\alpha, x_r^\alpha]$

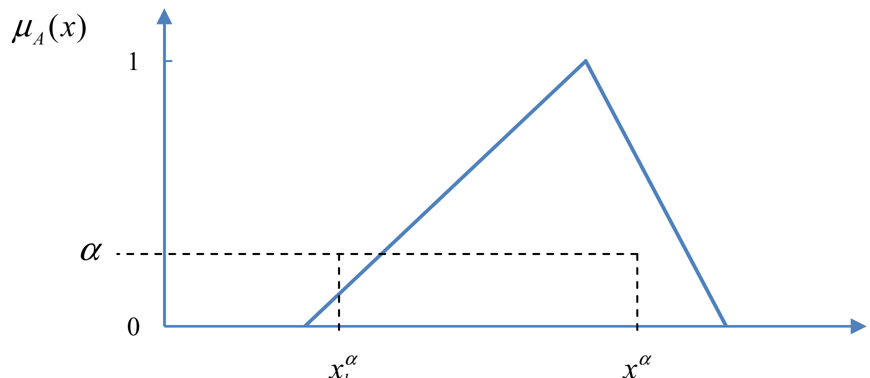


Figure 3. α -cut on Triangular Membership Function.

According to the definition, when α is close to 1, every element in subset A_α has a strong degree of membership. In this study, α -cut is adopted to represent the decision maker's level of confidence. The more confident the decision maker is, the larger α value is.

3.1.3 Pairwise Comparison with Fuzzy Synthetic Extent Analysis

As discussed above, considering the ambiguity and subjectivity of pairwise comparison, fuzzy logic is incorporated into this methodology. According to the definition of relative importance given in Table 1, a decision maker assigns a value of "3" to a pairwise comparison indicates he/she slightly favors one alternative to the other based on his/her experience. But due to the subjective nature of linguistic terms, this assessment can never be accurate or precise. Therefore, it is very important to incorporate fuzzy logic into pairwise comparisons.

The definition of triangular fuzzy numbers in Table 2 is used to facilitate the use of pairwise comparison. A triangular fuzzy number $\bar{a}$ means the decision maker thinks the importance ratio of two alternatives is "about *a*." In this study, the fuzzy numbers adopted to process relative importance ratios are defined in Table 2.

Table 2. Fuzzy Number Memberships of Pairwise Comparisons.

a	Fuzzy Number	*a*	Fuzzy Number
1	(1,1,2)	0.5	(0.33,0.5,1)
2	(1,2,3)	0.33	(0.25,0.33,0.5)
3	(2,3,4)	0.25	(0.2,0.25,0.33)
4	(3,4,5)	0.2	(0.17,0.2,0.25)
5	(4,5,6)	0.17	(0.14,0.17,0.2)
6	(5,6,7)	0.14	(0.13,0.14,0.17)
7	(6,7,8)	0.13	(0.11,0.13,0.14)
8	(7,8,9)	0.11	(0.11,0.11,0.13)
9	(8,9,9)	Diagonal Elements	(1,1,1)

In this study, all pairwise comparisons are processed as fuzzy numbers. The performance of the *i*th maintenance function with respect to the *j*th maintenance objective x_{ij} can be obtained through fuzzy extent analysis. [11]

$$x_{ij} = \frac{\sum_{s=1}^{n} \bar{a}_{ls}}{\sum_{l=1}^{n}\sum_{s=1}^{n} \bar{a}_{ls}}$$

where, $i = 1, 2, ..., n.$; $j = 1, 2, ..., m.$

3.1.4 Defuzzification based on Decision Maker's Attitude towards Risk

Since fuzzy numbers are not applicable in practice, those fuzzy numbers should be converted into quantitative and crisp results. The conversion is also called defuzzification process. There are many different defuzification methods available, such as center of area (COA), center of gravity (COG), fuzzy mean (FM), and weighted fuzzy mean (WFM). In this study, the proposed methodology incorporates decision maker's attitude towards risk into the defuzzification process.

When a decision maker has made an assessment and commits to it, he/she has made the decision and then uncertainties got involved in the process. And part of the uncertainties are systematically correlated with the decision maker's attitude toward risk: if the decision maker is a risk taker, he/she would intend to assign higher values to pairwise comparisons; if the decision maker is a risk averter, he/she would intend to assign lower values to pairwise comparisons.

One might use λ to represent a decision maker's attitude toward risk. Fuzzy number z after α-cut can be denoted as $z^{\alpha} = [z_l^{\alpha}, z_r^{\alpha}]$, and z^{α} is defuzzified through $z_{\alpha}^{\lambda} = \lambda z_r^{\alpha} + (1-\lambda) z_l^{\alpha}$. In this study, $\lambda = 1$ indicates the decision maker's attitude toward risk is optimistic (a risk taker); $\lambda = 0.5$ indicates the decision maker's attitude toward risk is moderate; and $\lambda = 0$ indicates the decision maker's attitude toward risk is pessimistic (a risk averter).

3.2 CONCEPTUAL FRAMEWORK

This report uses Multi-Attribute Analysis with Fuzzy Pairwise Comparison in prioritizing routine highway maintenance functions. The conceptual framework is shown in Figure 4. There are five main components in this methodology: 1) obtain the decision matrix of maintenance functions by fuzzy synthetic extent analysis; 2) obtain the fuzzy performances of all maintenance functions; 3) incorporate decision maker's degree of confidence into fuzzy performances; 4) incorporate decision maker's attitude toward risk into the maintenance function performances; 5) build a ranking index based on maintenance function performances, and rank the highway maintenance functions. The detailed discussion of these five components is given in this section.

3.2.1 Decision Matrix of Maintenance Functions

As discussed in Chapter 2, for any risk assessment process, the first step should be setting limits/scope of the analysis and identification of alternatives and objectives. For routine maintenance prioritization, the alternatives are the maintenance functions. But there are many types of maintenance functions in the field, thus the number of maintenance functions to be ranked should be limited, otherwise the problem would grow to an unmanageable size. Also, the maintenance objectives should be defined.

After maintenance functions are selected and maintenance objectives are clearly defined, a group of experts are asked to give their assessments on selected maintenance functions under the defined maintenance objectives, respectively. Those pairwise comparisons are processed as fuzzy numbers and denoted as $\bar{a}_{ls}$. The following matrix C_j consists of fuzzy pairwise comparisons of maintenance functions with respect to the jth maintenance objective.

$$C_j = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \dots & \bar{a}_{1n} \\ \bar{a}_{21} & \bar{a}_{22} & \dots & \bar{a}_{2n} \\ \dots & \dots & \dots & \dots \\ \bar{a}_{n1} & \bar{a}_{n2} & \dots & \bar{a}_{nn} \end{bmatrix} \tag{1}$$

$$\overline{a_{ls}} = \begin{cases} \bar{k} & l < s, k = 1, 2, \dots, 9 \\ 1 & l = s = 1, 2, \dots, n. \\ 1/\bar{k} & l > s \end{cases} \tag{2}$$

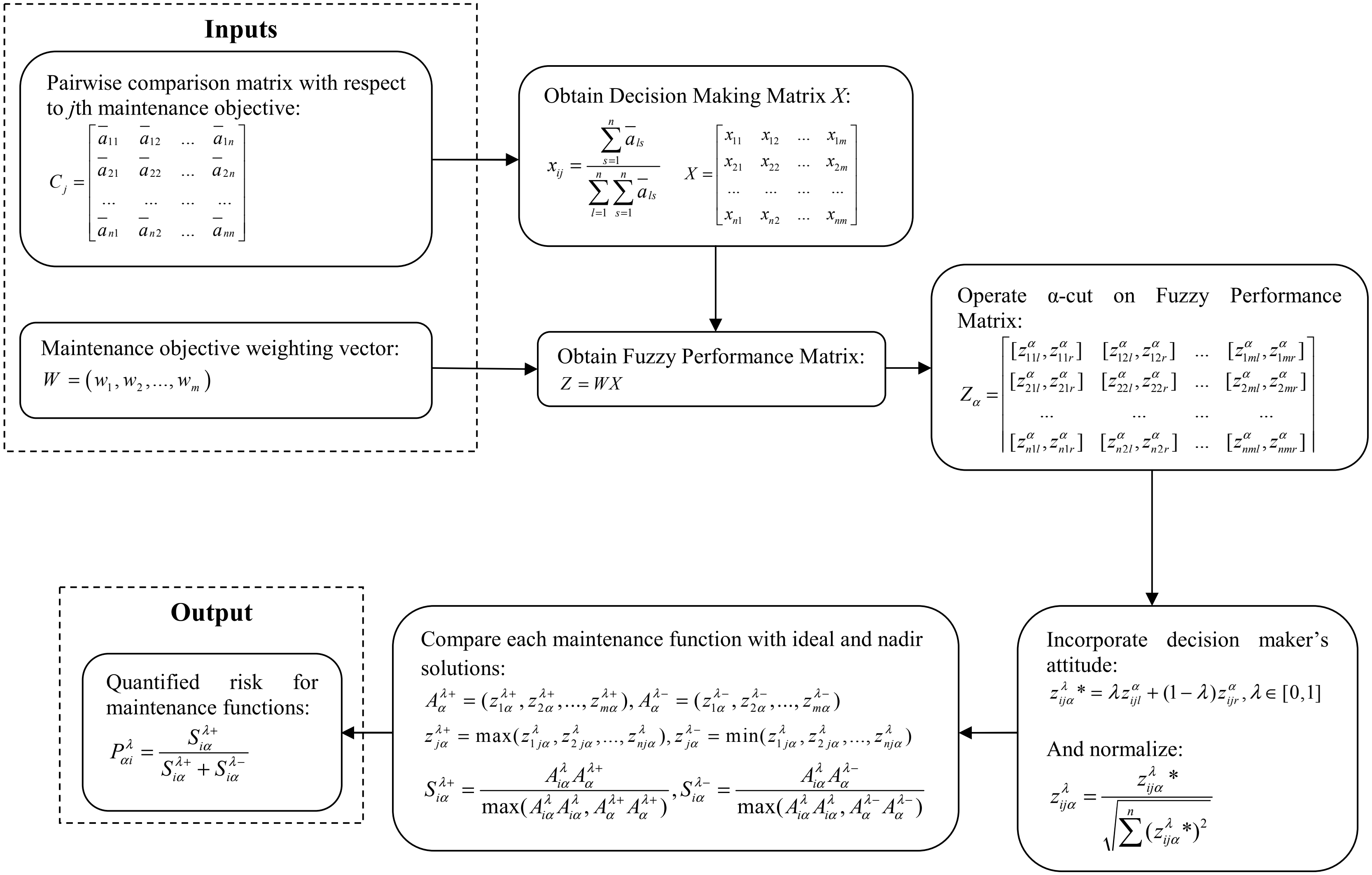


Figure 4. Conceptual Framework.

Fuzzy synthetic extent analysis is adopted to obtain the risk of not performing the *i*th maintenance function with respect to the *j*th maintenance objective. Then the same process is applied to all *n* maintenance functions under all *m* maintenance objectives. And then, the decision matrix *X* is obtained. It should be pointed out that each element x_{ij} in the matrix *X* is a fuzzy number.

$$x_{ij} = \frac{\sum_{s=1}^{n} \bar{a}_{ls}}{\sum_{l=1}^{n}\sum_{s=1}^{n} \bar{a}_{ls}} \tag{3}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \tag{4}$$

3.2.2 Fuzzy Performance of Maintenance Functions

Different maintenance objectives have different impacts on the total risk of deferring a specific maintenance function. In this study, the weights of maintenance objectives are obtained through a survey with a group of professionals with diverse backgrounds in highway maintenance. Denote the weighting vector as $W = (w_1, w_2, ..., w_m)$. Multiply the criteria weight vector *W* to decision matrix *X*, the overall risk matrix of the maintenance functions is shown below. Denote $w_j x_{ij}$ as z_{ij} which is also a fuzzy number.

$$Z = \begin{bmatrix} w_1 x_{11} & w_2 x_{12} & \dots & w_m x_{1m} \\ w_1 x_{21} & w_2 x_{22} & \dots & w_m x_{2m} \\ \dots & \dots & \dots & \dots \\ w_1 x_{n1} & w_2 x_{n2} & \dots & w_m x_{nm} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nm} \end{bmatrix} \tag{5}$$

3.2.3 Decision Maker's Degree of confidence

The α -cut represents a decision maker's degree of confidence in his/her assessments. The higher the α is, the more confident the decision maker is. In the extreme case, α equals to 1 when the decision maker is fully confident about his/her judgments. Given a confidence level α , the overall risk z_{ij} of the *i*th maintenance function after α -cut is a close interval $[z_{ijl}^{\alpha}, z_{ijr}^{\alpha}]$, where z_{ijl}^{α} is the left boundary of the α -cut, and z_{ijr}^{α} is the right boundary.

$$Z_{\alpha} = \begin{bmatrix} [z_{11l}^{\alpha}, z_{11r}^{\alpha}] & [z_{12l}^{\alpha}, z_{12r}^{\alpha}] & \dots & [z_{1ml}^{\alpha}, z_{1mr}^{\alpha}] \\ [z_{21l}^{\alpha}, z_{21r}^{\alpha}] & [z_{22l}^{\alpha}, z_{22r}^{\alpha}] & \dots & [z_{2ml}^{\alpha}, z_{2mr}^{\alpha}] \\ \dots & \dots & \dots & \dots \\ [z_{n1l}^{\alpha}, z_{n1r}^{\alpha}] & [z_{n2l}^{\alpha}, z_{n2r}^{\alpha}] & \dots & [z_{nml}^{\alpha}, z_{nmr}^{\alpha}] \end{bmatrix} \tag{6}$$

In the matrix Z_α, $[z_{ijl}^\alpha, z_{ijr}^\alpha]$ is the risk of not performing the *i*th maintenance function with respect to the *j*th maintenance objective. The width of this interval characterizes the decision maker's degree of confidence. The more confident the decision maker is, the smaller the interval will be. If the decision maker is fully confident, the interval would shrink to a point, which is the crisp case. However, when α is smaller than 1, the fuzzy numbers need to be defuzzified, in order to obtain an applicable crisp output.

3.2.4 Defuzzification based on Decision Maker's Attitude toward Risk

In the defuzzification process, λ is used to indicate the decision makers' attitudes toward risk. A λ of to 1, 0.5, or 0 indicates that the decision maker is optimistic, moderate, or pessimistic toward risk, respectively.

$$z_{ij\alpha}^{\lambda}* = \lambda z_{ijl}^{\alpha} + (1-\lambda) z_{ijr}^{\alpha} \tag{7}$$

The performance ratings are normalized as expressed in formula (8). And the overall risk matrix after normalization is shown in (9).

$$z_{ij\alpha}^{\lambda} = \frac{z_{ij\alpha}^{\lambda}*}{\sqrt{\sum_{i=1}^{n} (z_{ij\alpha}^{\lambda}*)^2}} \tag{8}$$

$$Z_\alpha^\lambda = \begin{bmatrix} z_{11\alpha}^\lambda & z_{12\alpha}^\lambda & \cdots & z_{1m\alpha}^\lambda \\ z_{21\alpha}^\lambda & z_{22\alpha}^\lambda & \cdots & z_{2m\alpha}^\lambda \\ \cdots & \cdots & \cdots & \cdots \\ z_{n1\alpha}^\lambda & z_{n2\alpha}^\lambda & \cdots & z_{nm\alpha}^\lambda \end{bmatrix} \tag{9}$$

The *i*th row in Z_α^λ is the risk vector $(z_{i1\alpha}^\lambda, z_{i2\alpha}^\lambda, \ldots, z_{im\alpha}^\lambda)$ of the *i*th maintenance function, with respect to all *m* maintenance objectives. The element in the *i*th row and *j*th column $z_{ij\alpha}^\lambda$ represents the risk rating of the *i*th maintenance function with respect to the *j*th maintenance function, when the decision maker's degree of confidence is α and attitude toward risk is λ.

3.2.5 Maintenance Function Prioritization

In the previous section, the (crisp) risk vector for each maintenance function is obtained through the defuzzification process. For each maintenance function, the risk vector $(z_{i1\alpha}^\lambda, z_{i2\alpha}^\lambda, \ldots, z_{im\alpha}^\lambda)$ has *m* elements; each representing the risk rating corresponding to one of the maintenance objectives. Under each maintenance objective, the maintenance functions can be ranked based on the risk ratings with respect to this specific objective. But considering all *m* maintenance objectives, the overall ranking of the maintenance functions cannot be easily obtained. Thus, after risk vectors are obtained, the next step would be constructing a consistent and effective overall ranking index.

There are several ways to build a ranking index. For example, the overall risk for a maintenance function could be evaluated by adding up the risk ratings with respect to all maintenance objectives. That is, use $\sum_{j=1}^{m} z_{ij\alpha}^\lambda$ as the overall ranking index for the *i*th maintenance function. And then rank the maintenance functions according to the descending order of the ranking index $\sum_{j=1}^{m} z_{ij\alpha}^{\lambda}$. As a result, the functions ranked higher in the list should be maintained first.

Also, the concept of similarity to the ideal solution and nadir solution could be used to construct the ranking index. The concept of ideal solution in the decision making process was first introduced by Zeleny [12] in his book "Multiple Criteria Decision Making." He suggests defining an "ideal solution" and using the similarity between the ideal solution and each alternative solution as the ranking index. The ideal solution in our routine maintenance function prioritization problem is a conceptual "maintenance function," of which the risk with respect to each maintenance objective is the highest among all maintenance functions. The overall risk vector of the ideal solution can be composed by selecting the highest value in each column in the overall risk matrix Z_{α}^{λ}. The similarity of the ideal solution and each maintenance function is then calculated, the one with highest similarity should be given the first priority.

Hwang and Yoon [13] extend the concept of "ideal solution" to "ideal solution and nadir solution." In their theory, the alternatives are not only compared with the positive ideal solution, they are also compared with the negative ideal solution. The negative ideal solution is also called the "nadir solution." In contrast to the positive ideal solution, the nadir solution in this study is a conceptual maintenance function with the lowest risks under all maintenance objectives. Combine the similarity with both ideal solution and nadir solution. The maintenance function closest to ideal solution and furthest to the nadir solution would be the first to be performed. The calculation process using similarity to ideal and nadir solution is expressed in formulas (10)-(13).

Under the jth maintenance objective, each maintenance function has a risk rating. The highest value $z_{j\alpha}^{\lambda+}$ and the lowest value $z_{j\alpha}^{\lambda-}$ are selected under this specific maintenance objective. The same process is applied to all m maintenance objectives, and the ideal solution $A_{\alpha}^{\lambda+}$, as well as the nadir solution $A_{\alpha}^{\lambda-}$ are therefore composed.

$$z_{j\alpha}^{\lambda+} = \max(z_{1j\alpha}^{\lambda}, z_{2j\alpha}^{\lambda}, ..., z_{nj\alpha}^{\lambda}), z_{j\alpha}^{\lambda-} = \min(z_{1j\alpha}^{\lambda}, z_{2j\alpha}^{\lambda}, ..., z_{nj\alpha}^{\lambda}). \quad (10)$$

$$A_{\alpha}^{\lambda+} = (z_{1\alpha}^{\lambda+}, z_{2\alpha}^{\lambda+}, ..., z_{m\alpha}^{\lambda+}), A_{\alpha}^{\lambda-} = (z_{1\alpha}^{\lambda-}, z_{2\alpha}^{\lambda-}, ..., z_{m\alpha}^{\lambda-}). \quad (11)$$

Here, $A_{\alpha}^{\lambda+}$ is the highest risk combination, representing the ideally most urgent maintenance function, while $A_{\alpha}^{\lambda-}$ is the lowest risk combination, representing the ideally least urgent maintenance function. The similarities of each maintenance function to $A_{\alpha}^{\lambda+}$ and $A_{\alpha}^{\lambda-}$ are evaluated. Then a performance index $P_{\alpha i}^{\lambda}$ is constructed to combine the similarity to both ideal and nadir solutions, as expressed in (13).

$$S_{i\alpha}^{\lambda+} = \frac{A_{i\alpha}^{\lambda} A_{\alpha}^{\lambda+}}{\max(A_{i\alpha}^{\lambda} A_{i\alpha}^{\lambda}, A_{\alpha}^{\lambda+} A_{\alpha}^{\lambda+})}$$

$$S_{i\alpha}^{\lambda-} = \frac{A_{i\alpha}^{\lambda} A_{\alpha}^{\lambda-}}{\max(A_{i\alpha}^{\lambda} A_{i\alpha}^{\lambda}, A_{\alpha}^{\lambda-} A_{\alpha}^{\lambda-})} \quad (12)$$

$$P_{\alpha i}^{\lambda} = \frac{S_{i\alpha}^{\lambda+}}{S_{i\alpha}^{\lambda+} + S_{i\alpha}^{\lambda-}}, i = 1, 2, ..., n. \quad (13)$$

The performance index $P_{\alpha i}^{\lambda}$ is essentially an indicator of the total risk exposed to the public and road users when the *i*th maintenance function is deferred or cancelled. Thus, maintenance functions could be ranked according to the descending order of $P_{\alpha i}^{\lambda}$, and the maintenance function with highest $P_{\alpha i}^{\lambda}$ value should be performed first.

CHAPTER 4. NUMERICAL EXAMPLE

In this chapter, a numerical example using Multi-Attribute Analysis with Fuzzy Pairwise Comparison on prioritizing highway maintenance functions is presented to illustrate an application of the methodology introduced in Chapter 3. This numerical case study uses collected data to prioritize routine maintenance functions according to the risk/penalty of not performing a specific type of maintenance project. By maintaining the most important maintenance functions, the total risk to the highway network is minimized under limited budget.

This chapter will first introduce the data acquisition process, including maintenance objective weighting, maintenance functions selection, and pairwise comparisons of the selected maintenance functions. To demonstrate fuzzy pairwise comparison analysis, an individual decision maker is randomly selected, and the data processing is demonstrated based on the pairwise comparison matrices obtained from this decision maker. Also, scenarios using different decision makers' degrees of confidence, different attitudes towards risk, and different fuzzy numbers are conducted, to help understand the uncertainty and subjectivity involved in the decision process.

4.1 Data Acquisition

Numerical analysis is necessary for testing the feasibility and effectiveness of the proposed methodology. In order to conduct the numerical analysis, a series of surveys is carried out. The surveys include the selection of maintenance functions and the maintenance objectives, maintenance objective weights, and pairwise comparisons of the selected maintenance functions. In this case study, the data is collected with the help of experts and engineers from TxDOT.

4.1.1 Maintenance Objectives

A workshop meeting was held on November 8, 2010, at TxDOT's Maintenance Division headquarters in Austin. A group of experts were asked to give their assessments and judgments on the maintenance objectives weighting of routine highway maintenance.

There are mainly two parts of the workshop concerning maintenance objectives. The first part is the selection of maintenance objectives. The experts were asked to nominate a number of maintenance objectives, and the top four objectives are adopted as the maintenance criteria in the following analysis. The selected maintenance objectives are safety, system preservation, aesthetics and system operation.

After the maintenance objectives are obtained, the weights of the selected maintenance objectives are to be determined. The same group of experts gave their estimates on the four selected maintenance objectives weighting. Each of the 10 experts assigned weights to the maintenance objectives in three rounds, each round using a different approach:

Round 1: Assign each objective a percentage to indicate the weight.

Round 2: Use 1.0 to indicate the lowest importance, and assume the importance scale among the objectives is linear.

Round 3: The importance of objectives should be ranked using a 1 to 5 scale, with 1 representing the least important and 5 representing the most important.

Table 3 through Table 5 are the results collected from experts at the workshop. In the "Participant Member" column, number 1 to 10 represent 10 individual experts, together with their judgments on maintenance objective weightings. The mean value, minimum value and maximum value of each maintenance objective weighting are also shown in the tables. Table 6 is the maintenance objective weighting used in this study: $W = (0.36, 0.32, 0.11, 0.21)$, where W is obtained by averaging the values obtained from the three rounds.

Table 3. Round 1 (Percentage Allocation).

Objectives	Participant Member										Mean	Min	Max	Relative Weights
	1	2	3	4	5	6	7	8	9	10				
Safety	40	45	30	35	30	35	40	20	30	35	34.0	20	45	0.34
System Preservation	30	20	40	30	30	30	25	55	40	25	32.5	20	55	0.32
Aesthetics	10	15	15	15	15	20	10	10	10	15	13.5	10	20	0.13
System Operation	20	20	15	20	25	15	25	15	30	25	21.0	15	30	0.21

Table 4. Round 2 (Incremental Ranking Base 1).

Objectives	Participant Member										Mean	Min	Max	Relative Weights
	1	2	3	4	5	6	7	8	9	10				
Safety	2.5	5	4	2	4	3	4	5	3	5	3.75	2	5	0.36
System Preservation	2	4	3	3	3	2.5	3	4	4	3	3.15	2	4	0.31
Aesthetics	1	1	1	1	1	1	1	1	1	2	1.10	1	2	0.11
System Operation	1.75	3	2	1	2	2	2	2	3	4	2.28	1	4	0.22

Table 5. Round 3 (Ranking: 1 = Least Important, 5 = Most Important).

Objectives	Participant Member										Mean	Min	Max	Relative Weights
	1	2	3	4	5	6	7	8	9	10				
Safety	5	5	5	5	5	4	5	5	4	5	4.80	4	5	0.37
System Preservation	4	3	4	4	4	5	4	4	5	4	4.10	3	5	0.32
Aesthetics	1	1	1	2	1	2	1	2	1	3	1.50	1	3	0.11
System Operation	2	2	3	4	3	1	3	3	3	2	2.60	1	4	0.20

Table 6. Maintenance Objective Weights.

Objectives	Safety	System Preservation	Aesthetics	System Operation
Relative Weights	0.36	0.32	0.11	0.21

4.1.2 Maintenance Functions Selection

Except for maintenance objectives weighting, the workshop also completed the selection of maintenance functions. The reason for selecting a limited number of maintenance functions is to assure this prioritization problem is manageable. TxDOT has more than 120 maintenance functions, but the number of frequently used maintenance functions is far less than 120. Also, ranking 120 alternatives based on pairwise comparisons would require a lot of work, and the problem would grow to an unmanageable size. So the basic logic of dealing with all 120 maintenance functions is to select a limited number of most frequently used maintenance functions as the alternatives to be ranked. The selected maintenance functions will be ranked based on the pairwise comparisons using the proposed methodology in Chapter 3.

In this study, 15 maintenance functions are selected. These 15 functions are from 3 maintenance categories, i.e., pavement related functions, roadside related functions, and traffic operation related functions. In each maintenance category, 5 maintenance functions are selected, and the rest of the maintenance functions are considered as less frequently used functions. The risks of unselected maintenance functions will be evaluated on a case-by-case basis. The selected maintenance functions are shown in Table 7.

Table 7. Selected Maintenance Functions.

Maintenance Category	Function No.	Maintenance Function
Pavement Related Functions	1	Seal Coat & Strip/Spot Seal
	2	Potholes, Semi-Permanent and Permanent Repair
	3	Leveling/Overlay
	4	Base Removal/Replacement/Base in Place Repair
	5	Edge Repair
Roadside Related Functions	6	Mowing & Spot Mowing
	7	Ditch Maint./Reshaping Ditches
	8	Hand & Chemical Vegetation
	9	Litter & Spot Litter
	10	Guard Fence
Traffic Operations Related Functions	11	Install/Reinstall Small, Large & Vandalized Signs
	12	High Performance Striping
	13	Maint. Of Isolated Coordinated Traffic Signals
	14	Paint & Bead Striping
	15	Illumination

With maintenance objectives and maintenance functions clearly defined, the hierarchical structure of the maintenance function prioritization process is therefore determined, as shown in Figure 5. There are two hierarchies: the first/higher hierarchy is maintenance objectives, and the second/lower hierarchy is the maintenance functions.

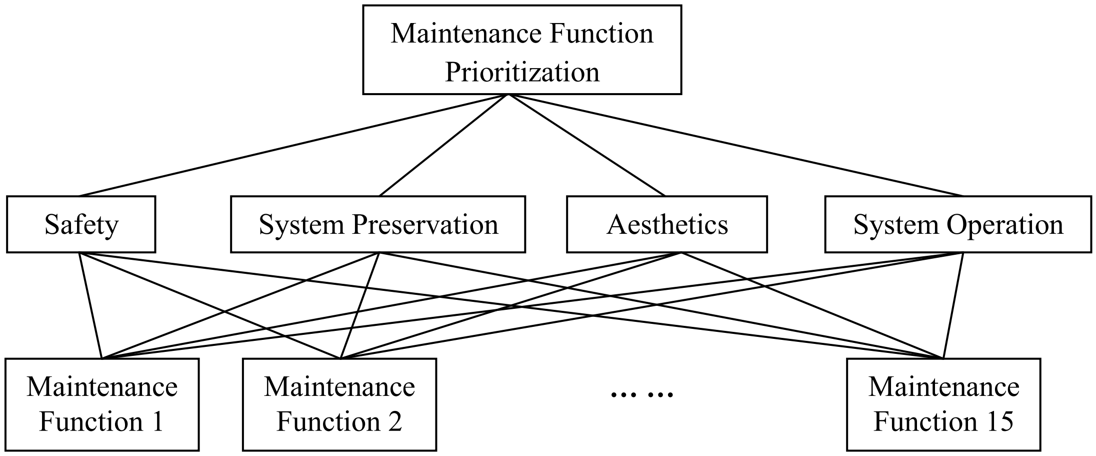


Figure 5. Hierarchical Structure of Maintenance Function Prioritization Process.

4.1.3 Pairwise Comparisons of Maintenance Functions

A group of experienced engineers in the Pharr TxDOT District in Texas were asked to give their judgments using pairwise comparisons of the selected 15 maintenance functions. This study is based on 11 correspondents. All of the 11 engineers gave pairwise comparisons of 15 maintenance functions with respect to the 4 selected maintenance objectives. The pairwise comparisons are based on 9-scale of relative importance defined in Table 1. And pairwise comparisons are processed as fuzzy numbers defined in Table 2.

To illustrate the fuzzy pairwise comparisons analysis, this study randomly selects one of the correspondences to demonstrate the computational process.

Function No.	1	2	3	…	15
1	1.00	0.20	0.33	…	0.50
2	5.00	1.00	2.00	…	3.00
3	3.00	0.50	1.00	…	2.00
…	…	…	…	…	…
15	2.00	0.33	0.50	…	1.00

This table is the pairwise comparison matrix under the maintenance objective “Safety.” In this matrix, every number represents the relative importance level between two maintenance functions, e.g. the first element in the third row is 3.00, which indicates this decision maker thinks from the safety aspect, Leveling/Overlay is “weakly important” over Seal Coat & Strip/Spot Seal, according to the definition in the 9-scale of the Relative Importance Table.

Chapter 3 discussed the necessity of introducing fuzzy logic into pairwise comparisons. Using the fuzzification rules defined in Table 2, substitute for the crisp numbers the fuzzy numbers as shown below.

Function No.	1	2	3	…	15
1	(1,1,1)	(0.17,0.2,0.25)	(0.25,0.33,0.50)	…	(0.33,0.5,1)
2	(4,5,6)	(1,1,1)	(1,2,3)	…	(2,3,4)
3	(2,3,4)	(0.33,0.5,0.1)	(1,1,1)	…	(1,2,3)
…	…	…	…	…	…
15	(1,2,3)	(0.25,0.33,0.50)	(0.33,0.5,0.1)	…	(1,1,1)

Use fuzzy extent analysis on the fuzzy pairwise comparison matrix under safety aspect. x_{11} in the following formula indicates the risk of not doing Maintenance Function 1 from the safety aspect.

Apply the same calculation to each row of the matrix, and repeat the same process to the other maintenance objectives, i.e. system preservation, aesthetics, and system operation. The

$$x_{11} = \frac{\sum_{s=1}^{15} \bar{a}_{ls}}{\sum_{l=1}^{15}\sum_{s=1}^{15} \bar{a}_{ls}}$$

$$= \frac{(1,1,1)+(0.17,0.2,0.25)+(0.25,0.33,0.50)+\ldots+(0.33,0.5,1)}{(1,1,1)+(0.17,0.2,0.25)+\ldots+(0.25,0.33,0.50)+\ldots+(1,2,3)+(0.25,0.33,0.50)+\ldots+(1,1,1)}$$

$$= (0.031, 0.058, 0.112)$$

decision matrix X from this individual decision maker is obtained.

$X =$

Safety	System Preservation	Aesthetics	System Operation
(0.031,0.058,0.112)	(0.077,0.134,0.221)	(0.050,0.102,0.199)	(0.052,0.105,0.193)
(0.071,0.140,0.248)	(0.071,0.129,0.222)	(0.009,0.018,0.040)	(0.056,0.113,0.213)
(0.044,0.087,0.170)	(0.075,0.137,0.232)	(0.047,0.100,0.202)	(0.073,0.137,0.244)
…	…	…	…
(0.024,0.048,0.101)	(0.022,0.041,0.086)	(0.032,0.064,0.142)	(0.023,0.045,0.100)

Since each criterion takes different weights in the multi-attribute decision making process, the overall risk matrix Z is calculated by multiplying X to weighting vector $W = (0.36, 0.32, 0.11, 0.21)$.

$Z =$

Safety	System Preservation	Aesthetics	System Operation
(0.011,0.021,0.40)	(0.025,0.043,0.071)	(0.005,0.011,0.022)	(0.011,0.022,0.041)
(0.026,0.050,0.089)	(0.023,0.041,0.071)	(0.001,0.002,0.004)	(0.012,0.024,0.045)
(0.016,0.031,0.061)	(0.024,0.044,0.074)	(0.005,0.011,0.022)	(0.015,0.029,0.051)
…	…	…	…
(0.009,0.017,0.036)	(0.007,0.013,0.028)	(0.003,0.007,0.016)	(0.005,0.010,0.021)

Let $\alpha = 0.6$ and $\lambda = 0.5$. Then the fuzzy performance matrix Z is defuzzified, with a specific degree of confidence $\alpha = 0.6$ and moderate attitude toward risk ($\lambda = 0.5$). Normalize the matrix to satisfy that, the summation of all 15 maintenance functions under each criterion is 1. Therefore the crisp normalized performance matrix Z_α^λ is obtained.

$$Z_{\alpha}^{\lambda} = \begin{vmatrix} \textit{Safety} & \textit{System Preservation} & \textit{Aesthetics} & \textit{System Operation} \\ 0.199 & 0.428 & 0.349 & 0.347 \\ 0.464 & 0.415 & 0.064 & 0.377 \\ 0.298 & 0.438 & 0.344 & 0.451 \\ \dots & \dots & \dots & \dots \\ 0.170 & 0.142 & 0.229 & 0.161 \end{vmatrix}$$

In each column in matrix Z_{α}^{λ}, select the highest value $z_{j\alpha}^{\lambda+}$ and the lowest value $z_{j\alpha}^{\lambda-}$. Then the performance index for each maintenance function can be calculated through formulas (10) through (13). The maintenance functions ranking is therefore determined as shown in Illustration 1.

It should be emphasized again that all the calculations above are based solely on the judgments from an individual decision maker. In this study, pairwise comparison data are collected from 11 engineers/decision makers, and the final performance index $P_{\alpha i}^{\lambda}$ for each maintenance function is the average of $P_{\alpha i}^{\lambda}$ from the 11 engineers/decision makers.

Illustration 1. An Example of Maintenance Function Ranking and Performance Indices.

Function No.	Maintenance Function	Performance Index	Ranking
1	Seal Coat & Strip/Spot Seal	0.823	4
2	Potholes, Semi-Permanent and Permanent Repair	0.850	3
3	Leveling/Overlay	0.860	1
4	Base Removal/Replacement/Base in Place Repair	0.852	2
5	Edge Repair	0.783	5
6	Mowing & Spot Mowing	0.410	14
7	Ditch Maint./Reshaping Ditches	0.473	13
8	Hand & Chemical Vegetation	0.314	15
9	Litter & Spot Litter	0.710	7
10	Guard Fence	0.670	8
11	Install/Reinstall Small, Large & Vandalized Signs	0.762	6
12	High Performance Striping	0.522	12
13	Maint. Of Isolated Coordinated Traffic Signals	0.653	9
14	Paint & Bead Striping	0.526	11
15	Illumination	0.563	10

4.2 Results Analysis

The decision maker's degree of confidence varies from case to case. The level of confidence represents how confident the decision maker is in his/her judgment. The decision maker's attitude toward risk indicates whether the decision maker is a risk taker or risk averter. These two parameters would impact the ranking indices and rankings of maintenance functions. To demonstrate the impact of different degrees of confidence and different attitudes toward risk on the ranking indices, a series of analyses is conducted.

4.2.1 Degree of Confidence

From the calculation demonstrated above, it is apparent that the parameters, such as degree of confidence α, decision makers' attitude toward risk λ, would impact the performance index $P_{\alpha i}^{\lambda}$, and therefore influence the final ranking of maintenance functions.

Operating α-cut on fuzzy performance matrix *Z* shortens the fuzzy subset interval of every element in *Z*. The fuzzy performance of *i*th maintenance function under *j*th objective after α-cut is $[z_{ijl}^{\alpha}, z_{ijr}^{\alpha}]$, where α reflects decision makers' degree of confidence in their judgments. If $\alpha = 0$, no changes have been made to the fuzzy performance matrix *Z* after α-cut operation; if $\alpha = 1$, fuzzy matrix *Z* is converted into a crisp matrix, in which every element is a crisp number. Apparently, the more confident the decision maker is, the higher the α value is, and the shorter the interval $[z_{ijl}^{\alpha}, z_{ijr}^{\alpha}]$ would be. That is to say, if a decision maker is not confident about his/her decision, the uncertainty of the decision will be comparatively high, and if the decision maker thinks he/she understands all the details and is fully confident about the judgments made, he/she will use a single crisp number instead of fuzzy numbers ($\alpha = 1$), and vice versa.

4.2.2 Decision Maker's Attitude toward Risk

Parameter λ represents decision maker's attitude toward risk. The λ value varies from 0 to 1. Decision maker's attitude toward risk indicates whether the decision maker tends to assign a higher value or a lower value to his/her judgments. In this study, three λ values are used: $\lambda = 0$, $\lambda = 0.5$ and $\lambda = 1$, corresponding to pessimistic decision maker, moderate decision maker and optimistic decision maker. Within each scenario, a series of analysis with different degrees of confidence is conducted.

4.2.2.1 Pessimistic Decision Maker

A value of 0 for λ means the decision maker's attitude toward risk is pessimistic. He/she is conservative about his/her assessments, i.e., the decision maker is a risk averter and tends to assign lower values to pairwise comparisons. Tables 8 and 9 show the performance indices and rankings of scenarios with different α values when the decision maker's attitude is pessimistic ($\lambda = 0$).

4.2.2.2 Moderate Decision Maker

A value of 0.5 for λ means the decision maker's attitude toward risk is moderate. He/she is neutral about his/her fuzzy assessments, i.e. the decision maker is neutral about the risk and tends to assign medium values to pairwise comparisons. Tables 10 and 11 show the performance indices and rankings of scenarios with different α values when the decision maker's attitude is moderate ($\lambda = 0.5$).

4.2.2.3 Optimistic Decision Maker

A value of 1 for λ means the decision maker's attitude toward risk is optimistic. He/she is more aggressive about his/her fuzzy assessment, i.e. the decision maker is a risk taker and tends to assign higher values to pairwise comparisons. Tables 12 and 13 show the performance indices and rankings of scenarios with different α values when the decision maker's attitude is optimistic ($\lambda = 1.0$).

Table 8. Performance Indices of Serial Cases with $\lambda = 0$ and Different α Values.

Function No.	Maintenance Function	$\lambda = 0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	0.566	0.569	0.565	0.573	0.586	0.596
2	Potholes, Semi-Permanent and Permanent Repair	0.632	0.635	0.632	0.639	0.649	0.657
3	Leveling/Overlay	0.639	0.642	0.644	0.646	0.653	0.659
4	Base Removal/Replacement/Base in Place Repair	0.616	0.618	0.620	0.621	0.627	0.632
5	Edge Repair	0.564	0.565	0.573	0.574	0.572	0.576
6	Mowing & Spot Mowing	0.402	0.400	0.412	0.408	0.392	0.388
7	Ditch Maint./Reshaping Ditches	0.463	0.462	0.461	0.458	0.457	0.454
8	Hand & Chemical Vegetation	0.399	0.397	0.391	0.385	0.385	0.378
9	Litter & Spot Litter	0.401	0.400	0.397	0.394	0.395	0.392
10	Guard Fence	0.601	0.602	0.598	0.597	0.605	0.607
11	Install/Reinstall Small, Large & Vandalized Signs	0.588	0.589	0.584	0.583	0.593	0.595
12	High Performance Striping	0.546	0.545	0.548	0.546	0.541	0.539
13	Maint. Of Isolated Coordinated Traffic Signals	0.568	0.568	0.575	0.574	0.571	0.573
14	Paint & Bead Striping	0.548	0.547	0.550	0.547	0.542	0.539
15	Illumination	0.546	0.545	0.552	0.548	0.537	0.533

Table 9. Maintenance Functions Ranking of Serial Cases with $\lambda = 0$ and Different α Values.

Function No.	Maintenance Function	$\lambda = 0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	7	6	8	8	6	5
2	Potholes, Semi-Permanent and Permanent Repair	2	2	2	2	2	2
3	Leveling/Overlay	1	1	1	1	1	1
4	Base Removal/Replacement/Base in Place Repair	3	3	3	3	3	3
5	Edge Repair	8	8	7	7	7	7
6	Mowing & Spot Mowing	13	13	13	13	14	14
7	Ditch Maint./Reshaping Ditches	12	12	12	12	12	12
8	Hand & Chemical Vegetation	15	15	15	15	15	15
9	Litter & Spot Litter	14	14	14	14	13	13
10	Guard Fence	4	4	4	4	4	4
11	Install/Reinstall Small, Large & Vandalized Signs	5	5	5	5	5	6
12	High Performance Striping	11	10	11	11	10	9
13	Maint. Of Isolated Coordinated Traffic Signals	6	7	6	6	8	8
14	Paint & Bead Striping	9	9	10	10	9	10
15	Illumination	10	11	9	9	11	11

Table 10. Performance Indices of Serial Cases with $\lambda = 0.5$ and Different α Values.

Function No.	Maintenance Function	$\lambda = 0.5$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	0.577	0.580	0.581	0.586	0.591	0.596
2	Potholes, Semi-Permanent and Permanent Repair	0.637	0.640	0.643	0.648	0.652	0.657
3	Leveling/Overlay	0.642	0.645	0.646	0.648	0.655	0.659
4	Base Removal/Replacement/Base in Place Repair	0.617	0.620	0.619	0.622	0.629	0.632
5	Edge Repair	0.562	0.564	0.570	0.572	0.573	0.576
6	Mowing & Spot Mowing	0.395	0.394	0.403	0.401	0.389	0.388
7	Ditch Maint./Reshaping Ditches	0.455	0.454	0.452	0.451	0.454	0.454
8	Hand & Chemical Vegetation	0.392	0.390	0.382	0.378	0.381	0.378
9	Litter & Spot Litter	0.395	0.395	0.390	0.388	0.393	0.392
10	Guard Fence	0.593	0.595	0.591	0.592	0.604	0.607
11	Install/Reinstall Small, Large & Vandalized Signs	0.580	0.582	0.577	0.578	0.592	0.595
12	High Performance Striping	0.537	0.537	0.539	0.538	0.539	0.539
13	Maint. Of Isolated Coordinated Traffic Signals	0.559	0.562	0.567	0.569	0.569	0.573
14	Paint & Bead Striping	0.539	0.539	0.541	0.540	0.539	0.539
15	Illumination	0.537	0.536	0.541	0.540	0.534	0.533

Table 11. Maintenance Functions Ranking of Serial Cases with $\lambda = 0.5$ and Different α Values.

Function No.	**Maintenance Function**	$\lambda = 0.5$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	6	6	5	5	6	5
2	Potholes, Semi-Permanent and Permanent Repair	2	2	2	2	2	2
3	Leveling/Overlay	1	1	1	1	1	1
4	Base Removal/Replacement/Base in Place Repair	3	3	3	3	3	3
5	Edge Repair	7	7	7	7	7	7
6	Mowing & Spot Mowing	14	14	13	13	14	14
7	Ditch Maint./Reshaping Ditches	12	12	12	12	12	12
8	Hand & Chemical Vegetation	15	15	15	15	15	15
9	Litter & Spot Litter	13	13	14	14	13	13
10	Guard Fence	4	4	4	4	4	4
11	Install/Reinstall Small, Large & Vandalized Signs	5	5	6	6	5	6
12	High Performance Striping	10	10	11	11	10	9
13	Maint. Of Isolated Coordinated Traffic Signals	8	8	8	8	8	8
14	Paint & Bead Striping	9	9	9	9	9	10
15	Illumination	11	11	10	10	11	11

Table 12. Performance Indices of Serial Cases with $\lambda = 1.0$ and Different α Values.

Function No.	Maintenance Function	$\lambda = 1.0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	0.613	0.607	0.603	0.600	0.598	0.596
2	Potholes, Semi-Permanent and Permanent Repair	0.651	0.653	0.654	0.656	0.657	0.657
3	Leveling/Overlay	0.653	0.655	0.657	0.658	0.659	0.659
4	Base Removal/Replacement/Base in Place Repair	0.619	0.623	0.627	0.629	0.631	0.632
5	Edge Repair	0.554	0.561	0.566	0.570	0.574	0.576
6	Mowing & Spot Mowing	0.370	0.375	0.379	0.383	0.385	0.388
7	Ditch Maint./Reshaping Ditches	0.423	0.433	0.440	0.446	0.450	0.454
8	Hand & Chemical Vegetation	0.368	0.371	0.373	0.375	0.377	0.378
9	Litter & Spot Litter	0.378	0.382	0.386	0.388	0.391	0.392
10	Guard Fence	0.563	0.578	0.588	0.596	0.602	0.607
11	Install/Reinstall Small, Large & Vandalized Signs	0.549	0.564	0.575	0.584	0.590	0.595
12	High Performance Striping	0.502	0.514	0.523	0.530	0.535	0.539
13	Maint. Of Isolated Coordinated Traffic Signals	0.529	0.543	0.554	0.562	0.568	0.573
14	Paint & Bead Striping	0.505	0.516	0.524	0.530	0.535	0.539
15	Illumination	0.502	0.512	0.519	0.525	0.530	0.533

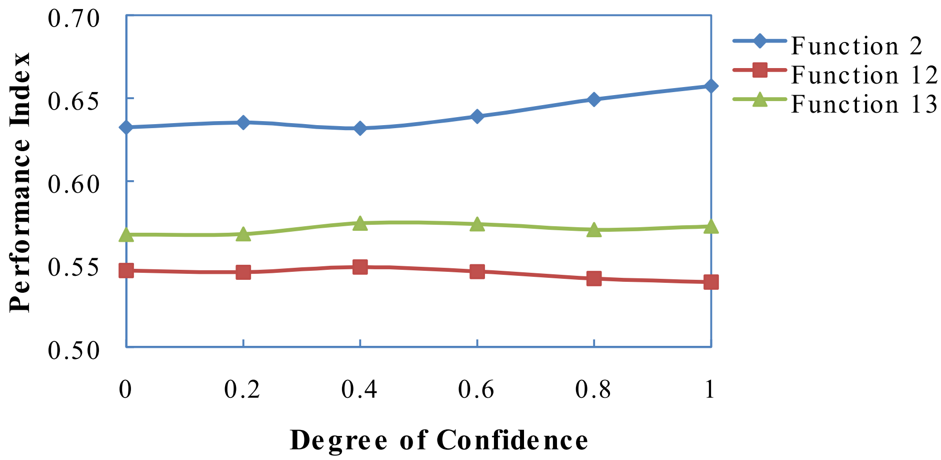
Table 13. Maintenance Functions Ranking of Serial Cases with $\lambda = 1.0$ and Different α Values.

Function No.	Maintenance Function	$\lambda = 1.0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	4	4	4	4	5	5
2	Potholes, Semi-Permanent and Permanent Repair	2	2	2	2	2	2
3	Leveling/Overlay	1	1	1	1	1	1
4	Base Removal/Replacement/Base in Place Repair	3	3	3	3	3	3
5	Edge Repair	6	7	7	7	7	7
6	Mowing & Spot Mowing	14	14	14	14	14	14
7	Ditch Maint./Reshaping Ditches	12	12	12	12	12	12
8	Hand & Chemical Vegetation	15	15	15	15	15	15
9	Litter & Spot Litter	13	13	13	13	13	13
10	Guard Fence	5	5	5	5	4	4
11	Install/Reinstall Small, Large & Vandalized Signs	7	6	6	6	6	6
12	High Performance Striping	10	10	10	10	9	9
13	Maint. Of Isolated Coordinated Traffic Signals	8	8	8	8	8	8
14	Paint & Bead Striping	9	9	9	9	10	10
15	Illumination	11	11	11	11	11	11

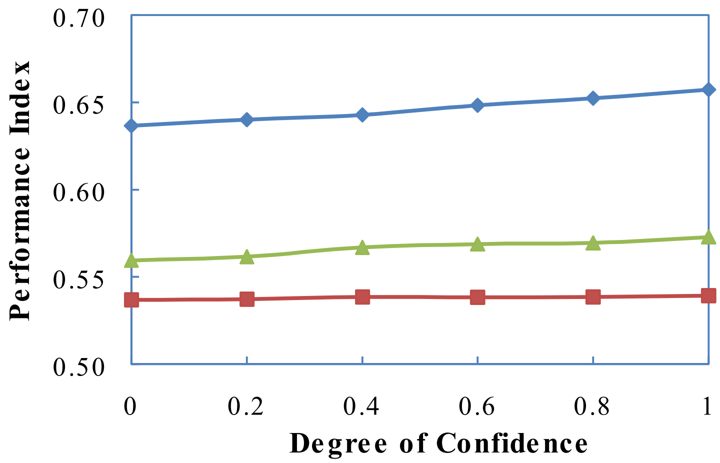
In the maintenance function ranking tables listed above, it is apparent that, regardless of the decision maker's degree of confidence and the decision maker's attitude toward risk, the overall ranking is stable across all scenarios. The top three maintenance functions are Leveling/Overlay, Potholes, Semi-Permanent and Permanent Repair and Base Removal/Replacement/Base in Place Repair, and the bottom three maintenance functions on the ranking list are Hand & Chemical Vegetation, Litter & Spot Litter, Mowing & Spot Mowing. According to the proposed methodology, the top three maintenance functions are the most urgent ones that should be ensured, and the bottom three maintenance functions should be considered to be deferred when there is a budget cut. The results are consistent with our common sense in practice.

It should be pointed out that the top three maintenance functions are all pavement related functions, since pavement related deficiencies generally have more direct impacts on the highway network, and are usually more hazardous. However, non-pavement related maintenance such as Guard Fence and Install/Reinstall Small, Large & Vandalized Signs also rank high in the list, prior to the rest of the pavement related maintenance functions. Since the performance index is essentially the quantified risk of not performing maintenance functions, the results prove that some non-pavement related functions are more important than some pavement related functions, further validating the concept of safety related maintenance to routine maintenance.

In order to further investigate the impact of different α values on performance indices, three maintenance functions - Function 2 (Potholes, Semi-Permanent and Permanent Repair), Function 12 (High Performance Striping) and Function 13 (Maint. Of Isolated Coordinated Traffic Signals) - are selected and compared in Figure 6, with different decision maker's attitudes.



(a) $\lambda = 0$



(b) $\lambda = 0.5$

(c) $\lambda = 1.0$

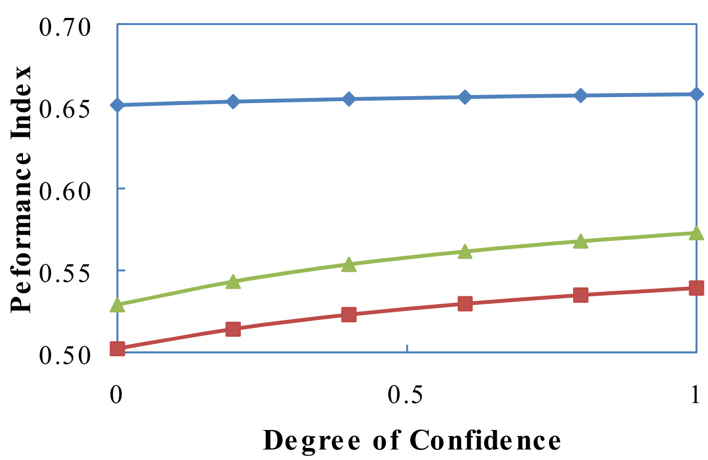


Figure 6. Performance Indices of Top Three Maintenance Functions.

Apparently, performance indices fluctuate as the degree of confidence changes under a fixed λ value (0; 0.5; 1.0). However, there are no consistant patterns for performance indices' fluctuations. Figure 7 represents the range of performance indices of the 15 maintenance functions, based on information provided in Tables 8 through 13. The dashed line in Figure 7 indicates the performance indices obtained under crisp scenario ($\alpha = 1$) using the proposed methodology; and the bars represent the ranges between maximum and minimum performance indices of each maintenance function. The range illustrates the variation, uncertainty and imprecision involved in the process.

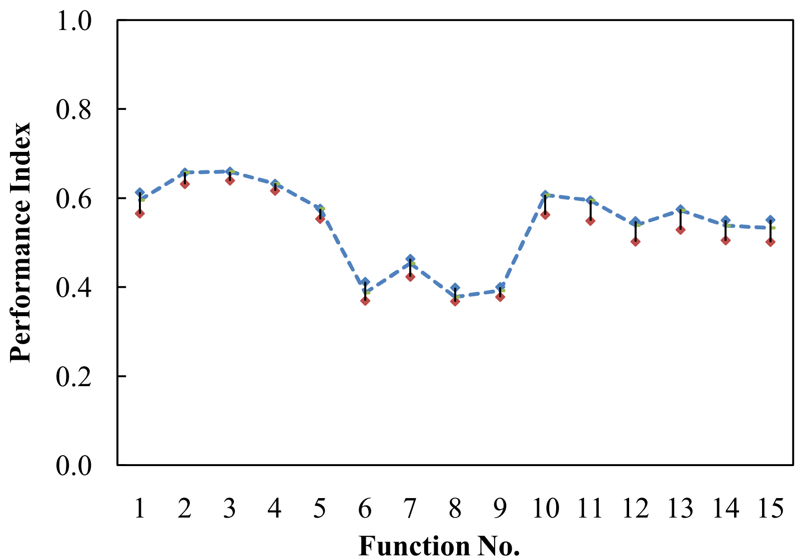


Figure 7. Performance Index Ranges of the Maintenance Functions.

4.2.3 Membership Function

Membership functions impact performance indices and rankings of the maintenance functions. In the previous analysis, the adopted membership functions are defined in Table 2, and the following analysis is based on a set of more dispersed membership functions, as shown in Table 14.

Comparing membership functions defined in Table 2 and Table 14, it is obvious that, with the same α values, using the membership functions in Table 14 would definitely result in a wider fuzzy performance interval $[z_{ijl}^{\alpha}, z_{ijr}^{\alpha}]$, i.e., this set of membership functions is more dispersed, and allowing more uncertainty in decision maker's judgments. A series of analyses using different λ and α values based on dispersed fuzzy numbers defined in Table 14 is shown in Tables 15 through 20. And Figure 8 shows the Performance Index-Degree of Confidence curves for Function 2, 12 and 13 using dispersed fuzzy numbers.

Table 14. Dispersed Fuzzy Numbers Membership Functions of Pairwise Comparisons.

a	Fuzzy Number	*a*	Fuzzy Number
1	(1,1,3)	0.5	(0.25,0.5,1)
2	(1,2,4)	0.33	(0.2,0.33,1)
3	(1,3,5)	0.25	(0.17,0.25,0.5)
4	(2,4,6)	0.2	(0.14,0.2,0.33)
5	(3,5,7)	0.17	(0.13,0.17,0.25)
6	(4,6,8)	0.14	(0.11,0.14,0.2)
7	(5,7,9)	0.13	(0.11,0.13,0.17)
8	(6,8,9)	0.11	(0.11,0.11,0.14)
9	(7,9,9)	Diagonal Elements	(1,1,1)

Table 15. Performance Indices of Serial Cases with $\lambda = 0$ and Different α Values Using Dispersed Fuzzy Numbers.

Function Function No.	Maintenance Function	$\lambda = 0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	0.566	0.569	0.565	0.573	0.586	0.596
2	Potholes, Semi-Permanent and Permanent Repair	0.632	0.635	0.632	0.639	0.649	0.657
3	Leveling/Overlay	0.639	0.642	0.644	0.646	0.653	0.659
4	Base Removal/Replacement/Base in Place Repair	0.616	0.618	0.620	0.621	0.627	0.632
5	Edge Repair	0.564	0.565	0.573	0.574	0.572	0.576
6	Mowing & Spot Mowing	0.402	0.400	0.412	0.408	0.392	0.388
7	Ditch Maint./Reshaping Ditches	0.463	0.462	0.461	0.458	0.457	0.454
8	Hand & Chemical Vegetation	0.399	0.397	0.391	0.385	0.385	0.378
9	Litter & Spot Litter	0.401	0.400	0.397	0.394	0.395	0.392
10	Guard Fence	0.601	0.602	0.598	0.597	0.605	0.607
11	Install/Reinstall Small, Large & Vandalized Signs	0.588	0.589	0.584	0.583	0.593	0.595
12	High Performance Striping	0.546	0.545	0.548	0.546	0.541	0.539
13	Maint. Of Isolated Coordinated Traffic Signals	0.568	0.568	0.575	0.574	0.571	0.573
14	Paint & Bead Striping	0.548	0.547	0.550	0.547	0.542	0.539
15	Illumination	0.546	0.545	0.552	0.548	0.537	0.533

Table 16. Maintenance Functions Ranking of Serial Cases with $\lambda = 0$ and Different α Values Using Dispersed Fuzzy Numbers.

Function No.	Maintenance Function	$\lambda = 0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	7	6	8	8	6	5
2	Potholes, Semi-Permanent and Permanent Repair	2	2	2	2	2	2
3	Leveling/Overlay	1	1	1	1	1	1
4	Base Removal/Replacement/Base in Place Repair	3	3	3	3	3	3
5	Edge Repair	8	8	7	7	7	7
6	Mowing & Spot Mowing	13	13	13	13	14	14
7	Ditch Maint./Reshaping Ditches	12	12	12	12	12	12
8	Hand & Chemical Vegetation	15	15	15	15	15	15
9	Litter & Spot Litter	14	14	14	14	13	13
10	Guard Fence	4	4	4	4	4	4
11	Install/Reinstall Small, Large & Vandalized Signs	5	5	5	5	5	6
12	High Performance Striping	11	10	11	11	10	9
13	Maint. Of Isolated Coordinated Traffic Signals	6	7	6	6	8	8
14	Paint & Bead Striping	9	9	10	10	9	10
15	Illumination	10	11	9	9	11	11

Table 17. Performance Indices of Serial Cases with $\lambda = 0.5$ and Different α Values Using Dispersed Fuzzy Numbers.

Function No.	Maintenance Function	$\lambda = 0.5$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	0.577	0.580	0.581	0.586	0.591	0.596
2	Potholes, Semi-Permanent and Permanent Repair	0.637	0.640	0.643	0.648	0.652	0.657
3	Leveling/Overlay	0.642	0.645	0.646	0.648	0.655	0.659
4	Base Removal/Replacement/Base in Place Repair	0.617	0.620	0.619	0.622	0.629	0.632
5	Edge Repair	0.562	0.564	0.570	0.572	0.573	0.576
6	Mowing & Spot Mowing	0.395	0.394	0.403	0.401	0.389	0.388
7	Ditch Maint./Reshaping Ditches	0.455	0.454	0.452	0.451	0.454	0.454
8	Hand & Chemical Vegetation	0.392	0.390	0.382	0.378	0.381	0.378
9	Litter & Spot Litter	0.395	0.395	0.390	0.388	0.393	0.392
10	Guard Fence	0.593	0.595	0.591	0.592	0.604	0.607
11	Install/Reinstall Small, Large & Vandalized Signs	0.580	0.582	0.577	0.578	0.592	0.595
12	High Performance Striping	0.537	0.537	0.539	0.538	0.539	0.539
13	Maint. Of Isolated Coordinated Traffic Signals	0.559	0.562	0.567	0.569	0.569	0.573
14	Paint & Bead Striping	0.539	0.539	0.541	0.540	0.539	0.539
15	Illumination	0.537	0.536	0.541	0.540	0.534	0.533

Table 18. Maintenance Functions Ranking of Serial Cases with $\lambda = 0.5$ and Different α Values Using Dispersed Fuzzy Numbers.

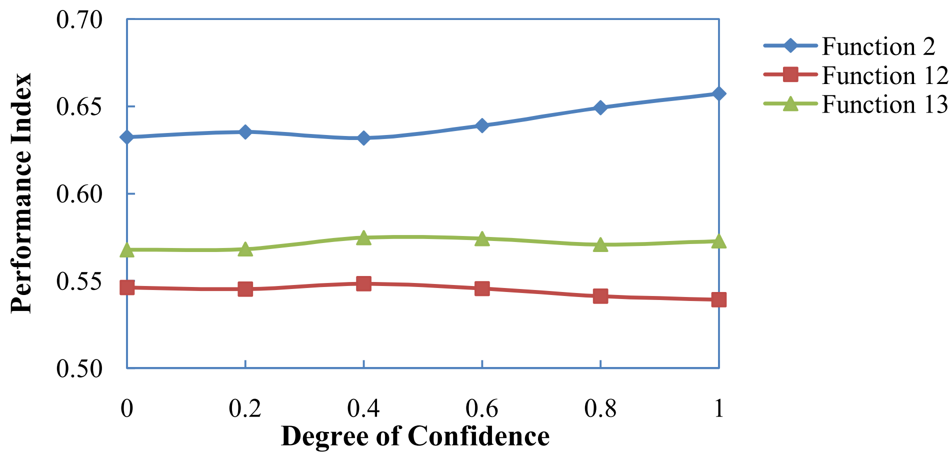
Function No.	Maintenance Function	$\lambda = 0.5$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	6	6	5	5	6	5
2	Potholes, Semi-Permanent and Permanent Repair	2	2	2	2	2	2
3	Leveling/Overlay	1	1	1	1	1	1
4	Base Removal/Replacement/Base in Place Repair	3	3	3	3	3	3
5	Edge Repair	7	7	7	7	7	7
6	Mowing & Spot Mowing	14	14	13	13	14	14
7	Ditch Maint./Reshaping Ditches	12	12	12	12	12	12
8	Hand & Chemical Vegetation	15	15	15	15	15	15
9	Litter & Spot Litter	13	13	14	14	13	13
10	Guard Fence	4	4	4	4	4	4
11	Install/Reinstall Small, Large & Vandalized Signs	5	5	6	6	5	6
12	High Performance Striping	10	10	11	11	10	9
13	Maint. Of Isolated Coordinated Traffic Signals	8	8	8	8	8	8
14	Paint & Bead Striping	9	9	9	9	9	10
15	Illumination	11	11	10	10	11	11

Table 19. Performance Indices of Serial Cases with $\lambda = 1.0$ and Different α Values Using Dispersed Fuzzy Numbers.

Function No.	Maintenance Function	$\lambda = 1.0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	0.613	0.607	0.603	0.600	0.598	0.596
2	Potholes, Semi-Permanent and Permanent Repair	0.651	0.653	0.654	0.656	0.657	0.657
3	Leveling/Overlay	0.653	0.655	0.657	0.658	0.659	0.659
4	Base Removal/Replacement/Base in Place Repair	0.619	0.623	0.627	0.629	0.631	0.632
5	Edge Repair	0.554	0.561	0.566	0.570	0.574	0.576
6	Mowing & Spot Mowing	0.370	0.375	0.379	0.383	0.385	0.388
7	Ditch Maint./Reshaping Ditches	0.423	0.433	0.440	0.446	0.450	0.454
8	Hand & Chemical Vegetation	0.368	0.371	0.373	0.375	0.377	0.378
9	Litter & Spot Litter	0.378	0.382	0.386	0.388	0.391	0.392
10	Guard Fence	0.563	0.578	0.588	0.596	0.602	0.607
11	Install/Reinstall Small, Large & Vandalized Signs	0.549	0.564	0.575	0.584	0.590	0.595
12	High Performance Striping	0.502	0.514	0.523	0.530	0.535	0.539
13	Maint. Of Isolated Coordinated Traffic Signals	0.529	0.543	0.554	0.562	0.568	0.573
14	Paint & Bead Striping	0.505	0.516	0.524	0.530	0.535	0.539
15	Illumination	0.502	0.512	0.519	0.525	0.530	0.533

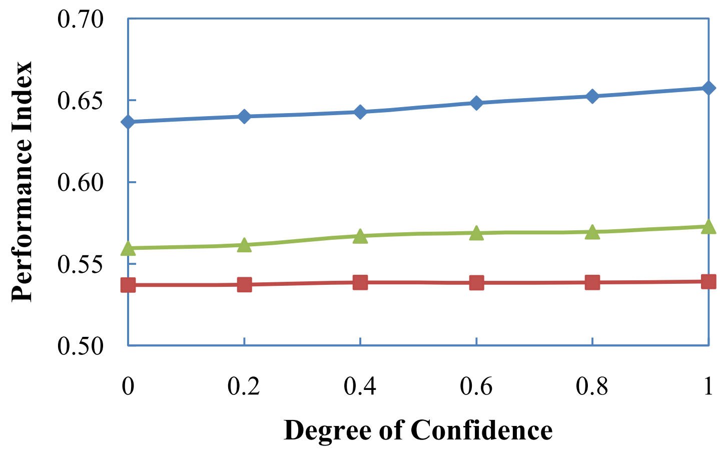
Table 20. Maintenance Functions Ranking of Serial Cases with $\lambda = 1.0$ and Different α Values Using Dispersed Fuzzy Numbers.

Function No.	Maintenance Function	$\lambda = 1.0$					
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
1	Seal Coat & Strip/Spot Seal	4	4	4	4	5	5
2	Potholes, Semi-Permanent and Permanent Repair	2	2	2	2	2	2
3	Leveling/Overlay	1	1	1	1	1	1
4	Base Removal/Replacement/Base in Place Repair	3	3	3	3	3	3
5	Edge Repair	6	7	7	7	7	7
6	Mowing & Spot Mowing	14	14	14	14	14	14
7	Ditch Maint./Reshaping Ditches	12	12	12	12	12	12
8	Hand & Chemical Vegetation	15	15	15	15	15	15
9	Litter & Spot Litter	13	13	13	13	13	13
10	Guard Fence	5	5	5	5	4	4
11	Install/Reinstall Small, Large & Vandalized Signs	7	6	6	6	6	6
12	High Performance Striping	10	10	10	10	9	9
13	Maint. Of Isolated Coordinated Traffic Signals	8	8	8	8	8	8
14	Paint & Bead Striping	9	9	9	9	10	10
15	Illumination	11	11	11	11	11	11



(a) $\lambda = 0$

(b) $\lambda = 0.5$



(c) $\lambda = 1.0$

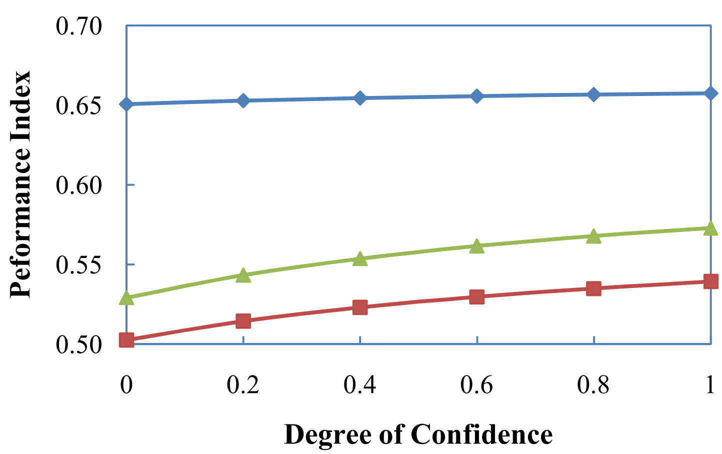


Figure 8. Performance Indices of Top Three Maintenance Functions Using Dispersed Fuzzy Numbers.

In order to demonstrate the impact of different fuzzy numbers on maintenance function performance indices, performance indices of Function 2 and Function 12 using dispersed and un-dispersed fuzzy numbers from moderate decision maker assessments (i.e. $\lambda = 0.5$) are illustrated in Figure 9 as an example.

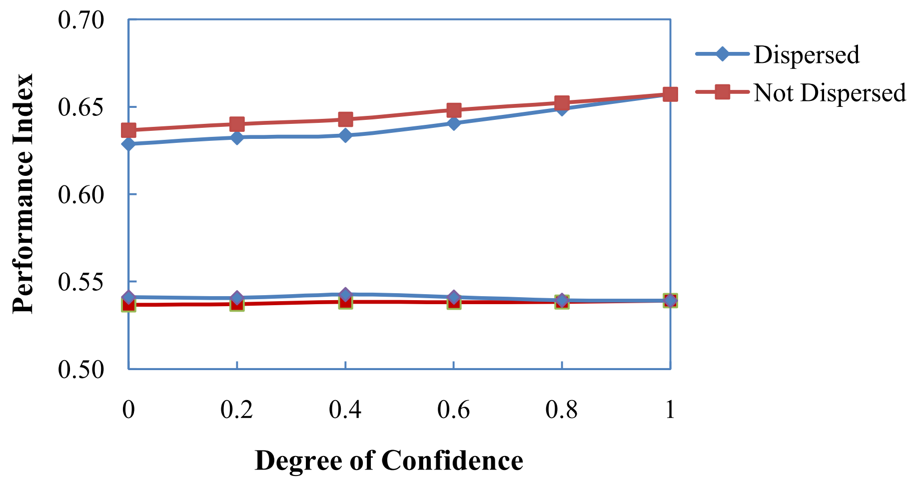


Figure 9. Comparison between Different Fuzzy Numbers.

In Figure 9, it is apparent that performance indices based on dispersed fuzzy numbers have larger differentials between scenarios using different degrees of confidence α than performance indices based on un-dispersed fuzzy numbers. The larger differentials indicate a higher variation. The fluctuations reflect the complexity of the decision process and other external influencing factors that are not controlled by the decision makers. The magnitude of fluctuations in rankings and performance indices basically depend on the selection of fuzzy membership functions. A set of more dispersed fuzzy membership functions would lead to a larger magnitude in the fluctuations.

However, the fuzzy numbers are preselected, and they reflect how complicated or indeterminate the decision environment is. In other words, degree of confidence α can vary from person to person, but fuzzy numbers are predetermined and universal across the entire calculation process.

CHAPTER 5. COMPARISON WITH CRISP AHP APPROACH

The collected data could also be processed using AHP for prioritizing highway maintenance functions. The proposed methodology would regress to crisp scenarios when the decision maker's degree of confidence is 1. Theoretically, the outcomes from regressed Multi-Attribute Analysis with Fuzzy Pairwise Comparison should be close to the results from crisp AHP. This chapter will present the results from the crisp AHP approach, and compare the outcome with results using Multi-Attribute Analysis with Fuzzy Pairwise Comparison.

5.1 CRISP AHP APPROACH

As mentioned in Chapter 2, AHP is a popular methodology in prioritizing projects. In this chapter, the same 15 maintenance functions are evaluated under the same set of objectives, the same weighting vector is used, and the analysis is based on the same set of pairwise comparison data as used in fuzzy pairwise comparison approach. The calculation process is briefly demonstrated as follows. [8]

a_{ij} is the pairwise assessment set obtained from decision makers. For $a_{ii} = 1$ and $a_{ij} = 1/\ a_{ji}$ for all $i, j \leq n$, $l \leq m$. n=15 is the number of maintenance functions, and m=4 is the number of objectives. A_l is the pairwise comparison matrix, which contains pairwise comparisons of all 15 maintenance functions with respect to the lth objective.

$$A_l = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2n} \\ \dots & \dots & 1 & \dots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \dots & 1 \end{bmatrix}$$

where, a_{ij} is the importance scale of alternative i over alternative j;

a_{ji} is the importance scale of alternative j over alternative i.

$$A_l X_l' = \lambda_{\max}^l X_l'$$

where, $\lambda_{\max}^l$ is the principal eigenvalue (largest eigenvalue of matrix A_l);

X_l is the priority vector of the maintenance functions under the lth objective.

The results obtained from the above computations are the priorities or rankings of the maintenance functions with respect to different objectives. Once this process has been performed on all $A_l (l = 1, 2, 3, 4.)$, multiply the priority vector of the maintenance functions X with its corresponding objective weights and obtain V.

$$V=WX$$

where, $V = (v_1, v_2, ..., v_n)$ is the weighting vector of all n alternatives;

$W = (w_1, w_2, ..., w_m)$ is the weighting vector of objectives, $w_l \ (l = 1, 2, ..., m)$ is the weight assigned to the lth objective;

$X = (X_1, X_2, ..., X_m)'$, and $X_l = (X_{l1}, X_{l2}, ..., X_{ln})$ is the maintenance functions weights with respect to the *l*th objective.

$X = (X_1, X_2, ..., X_m)'$ is the relative weights matrix of all the maintenance functions with respect to each of the *m* objectives. $W = (w_1, w_2, ..., w_m)$ is the maintenance objectives weighting vector. *ORW* denotes the Overall Relative Weight of each maintenance activity, which is essentially the element v_i in the overall priority vector $V = (v_1, v_2, ..., v_n)$. Therefore, for maintenance function *i*, the overall relative weight is,

$$ORW_i = WX = \sum_{l=1}^{m} X_{li}$$

ORW is essentially the ranking index in AHP. Rank the maintenance functions according to the descending order of *ORW*. The maintenance function with the highest priority value ranks first, and the lowest ranks as the last function to be considered.

Apply crisp AHP to the maintenance functions pairwise comparison data and maintenance objective weighting vector $W = (0.36, 0.32, 0.11, 0.21)$ obtained from TxDOT, and then obtain the final *ORW*s of the 15 maintenance functions. The rankings and *ORW* of each of the 15 maintenance functions are shown in Table 21.

Table 21. Individual Maintenance Activity Weights.

Ranking	Maintenance Function	Safety	System Preservation	System Operation	Aesthetics	ORW
7	Seal Coat & Strip/Spot Seal	0.0691	0.0860	0.0592	0.0573	0.0709
1	Potholes, Semi-Permanent and Permanent Repair	0.0908	0.0966	0.0755	0.0522	0.0829
2	Leveling/Overlay	0.0703	0.0977	0.0813	0.0817	0.0827
3	Base Removal/Replacement/Base in Place Repair	0.0728	0.0963	0.0800	0.0537	0.0771
10	Edge Repair	0.0716	0.0702	0.0594	0.0474	0.0647
14	Mowing & Spot Mowing	0.0441	0.0539	0.0479	0.0597	0.0509
12	Ditch Maint./Reshaping Ditches	0.0519	0.0562	0.0501	0.0507	0.0528
15	Hand & Chemical Vegetation	0.0422	0.0524	0.0428	0.0621	0.0497
13	Litter & Spot Litter	0.0379	0.0458	0.0461	0.0871	0.0517
5	Guard Fence	0.0888	0.0564	0.0679	0.0742	0.0731
6	Install/Reinstall Small, Large & Vandalized Signs	0.0755	0.0570	0.0835	0.0835	0.0722
11	High Performance Striping	0.0672	0.0486	0.0740	0.0685	0.0622
4	Maint. Of Isolated Coordinated Traffic Signals	0.0799	0.0671	0.0853	0.0733	0.0750
9	Paint & Bead Striping	0.0665	0.0565	0.0738	0.0722	0.0653
8	Illumination	0.0715	0.0593	0.0734	0.0765	0.0689

5.2 COMPARISON AND DISCUSSION

For the proposed Multi-Attribute Analysis with Fuzzy Pairwise Comparison methodology described in Chapter 3, if the degree of confidence $\alpha = 1$, regardless of the λ value, the fuzzy performance interval after α cut $[z_{ijl}^{\alpha}, z_{ijr}^{\alpha}]$ would shrink to a single point. Thus the fuzzy analysis would regress to a crisp scenario. Theoretically, the $\alpha = 1$ scenario is closest to the results from crisp AHP process. However, the results are not identical to each other, since the ranking indices in these two methodologies are constructed using different approaches. For a better comparison, rescale the performance indices of all 15 maintenance functions obtained from fuzzy approach to let the summation of all performance indices equal 1. The ranking indices and maintenance function rankings from these two methodologies are compared in Table 22.

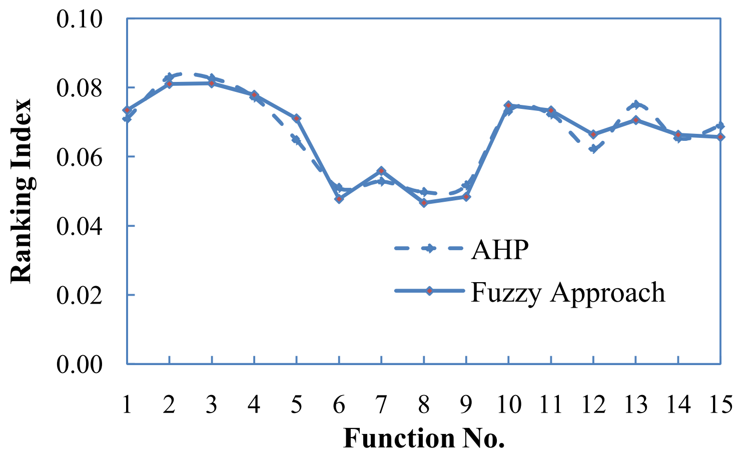


Figure 10. Comparison of Results obtained from AHP and Fuzzy Approach.

There are some differences in rankings obtained from the two approaches, e.g., Leveling/Overlay ranks No.1 in the fuzzy approach, but in crisp AHP, "Potholes, Semi-Permanent and Permanent Repair" ranks even higher than Leveling/Overlay. However, the ranking indices of these two maintenance functions are very close in these two methodologies. Figure 10 illustrates the ranking indices obtained from both crisp AHP and Multi-Attribute Analysis with Fuzzy Pairwise Comparison. This comparison seems to validate the effectiveness of prioritizing highway maintenance functions using Multi-Attribute Analysis with Fuzzy Pairwise Comparison.

Table 22. Comparison of Ranking Indices and Rankings from Crisp AHP and Fuzzy Approach.

Function No.	Maintenance Function	Crisp AHP		Fuzzy Approach	
		ORW	Ranking	Rescaled Performance Index	Ranking
1	Seal Coat & Strip/Spot Seal	0.0709	7	0.0734	5
2	Potholes, Semi-Permanent and Permanent Repair	0.0829	1	0.0810	2
3	Leveling/Overlay	0.0827	2	0.0812	1
4	Base Removal/Replacement/Base in Place Repair	0.0771	3	0.0779	3
5	Edge Repair	0.0647	10	0.0710	7
6	Mowing & Spot Mowing	0.0509	14	0.0477	14
7	Ditch Maint./Reshaping Ditches	0.0528	12	0.0559	12
8	Hand & Chemical Vegetation	0.0497	15	0.0466	15
9	Litter & Spot Litter	0.0517	13	0.0483	13
10	Guard Fence	0.0731	5	0.0748	4
11	Install/Reinstall Small, Large & Vandalized Signs	0.0722	6	0.0733	6
12	High Performance Striping	0.0622	11	0.0664	9
13	Maint. Of Isolated Coordinated Traffic Signals	0.0750	4	0.0705	8
14	Paint & Bead Striping	0.0653	9	0.0663	9
15	Illumination	0.0689	8	0.0657	11

CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

This report introduces the methodology of prioritizing routine maintenance functions using Multi-Attribute Analysis with Fuzzy Pairwise Comparison. A case study is conducted with a series of scenarios using different degrees of confidence and attitudes towards risk. Also, the results are compared with the outcome from crisp AHP using the same data set. Through the analysis, the validity of the proposed methodology is proved. With conclusions presented in this chapter, recommendations are also given to provide guidance to decision makers.

6.1 CONCLUSIONS

This study proposes a methodology of using Multi-Attribute Decision Making with Fuzzy Pairwise Comparison for solving maintenance prioritization problems. The validity of the proposed methodology is demonstrated. Prioritizing routine maintenance functions using Multi-Attribute Analysis with Fuzzy Pairwise Comparison enables decision makers to use parameters to depict different degrees of confidence in the assessments, and decision makers' different attitudes towards risk. Moreover, by using different sets of fuzzy membership functions, this methodology enables decision makers to capture the complexity and unpredictability of the decision environment.

Despite the simplicity and comprehensibility of the proposed approach, this study shows its effectiveness and efficiency for prioritizing routine highway maintenance functions. This methodology enables decision makers to quantify and minimize the total risk due to cancelling or deferring maintenance functions during budget cuts. This methodology assists decision makers in evaluating the total risk of current work plans and rescheduling maintenance activities to minimize the risks induced by budget fluctuations if needed. And therefore, the proposed methodology can potentially help state DOTs in revising their work plans to accommodate budget fluctuations.

6.2 RECOMMENDATIONS

It is recommended that decision makers conduct additional analysis before determining decision maker's degree of confidence and attitude towards risk. If the results from the analysis show a low level of fluctuations in maintenance rankings as degrees of confidence or attitudes toward risk change, pairwise comparison assessments could be simply processed as crisp numbers instead of conducting fuzzy analysis. If the fluctuations and variations are relatively high, decision maker's degree of confidence and attitude toward risk should be carefully acquired and included.

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