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16. Abstract

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OPTIMAL INFRASTRUCTURE MAINTENANCE SCHEDULING PROBLEM UNDER BUDGET UNCERTAINTY

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ABSTRACT

This research addresses a general class of infrastructure asset management problems. Infrastructure agencies usually face budget uncertainties that will eventually lead to suboptimal planning if maintenance decisions are made without taking the uncertainty into consideration. It is important for decision makers to adopt maintenance scheduling policies that take future budget uncertainty into consideration. The author proposes a multistage, stochastic linear programming model to address this problem. The author also develops solution procedures using the augmented Lagrangian decomposition algorithm and scenario reduction method. A case study exploring the computational characteristics of the proposed methods is conducted and the benefit of using the stochastic programming approach is discussed.

EXECUTIVE SUMMARY

An efficient transportation infrastructure network is vital to economic and social development. Infrastructure maintenance consumes a significant proportion of the surface transportation budget, while the costs borne by the road-using public for vehicle operation and depreciation are even greater. Facilities must be designed and constructed with budget and other applicable constraints. Infrastructure maintenance management is one of the most important components of infrastructure management. It is the process of developing alternative maintenance strategies and determining the best solution to ensure desired level of service. For many types of infrastructure facilities, the service life can be extended beyond the original design life by applying maintenance treatments. Maintenance strategies are generally considered a sequence of treatments selected from a list of possible treatments generally available for the facility. Life-cycle cost concepts should be used to determine the difference in costs between various strategies. Costs should consider those borne by both user and agency.

Maintenance options of an infrastructure facility consist of various routine, preventive, or reactive activities, and other rehabilitation and replacement techniques. Maintenance expenditure is one of the costly infrastructure investments. From a mathematical point of view, there are two types of maintenance scheduling problems. The first one is network-level problem, where decision makers face great challenges of determining which facility is to be repaired, when and how repairs should be carried out, and what treatment to use. Another is the project-level maintenance problem, in which only the maintenance scheduling of one facility is considered.

There are other uncertainties in the infrastructure management process. For example, infrastructure deterioration is a dynamic, complicated, and stochastic process affected by a variety of factors such as usage, environmental conditions, and structural capacities, as well as certain unobserved factors. Hence, the performance of an infrastructure facility can never be predicted with absolute certainty. Ignoring such uncertainties during the modeling process may compromise the validity of an optimal solution. It is also important to take those uncertainties into consideration when making maintenance resource allocation decisions.

A road network case is studied as part of this research. The following findings indicate that the proposed model and solution procedure is able to solve the maintenance scheduling problem efficiently and effectively. The benefit of using the stochastic programming approach over a deterministic approach is also discussed. Stochastic programming solutions, which take future budget uncertainty into consideration, tend to allocate more resource into preventive maintenance than deterministic solution that ignores the uncertainty information. The proposed methodology can help decision makers effectively obtain optimal maintenance planning under budget uncertainty.

The objective of this research is to develop a network-level infrastructure maintenance scheduling problem under budget uncertainty. The problem was formulated as a multistage, linear stochastic programming model. The proposed model differs from its deterministic counterpart in that it attempts to find the optimal maintenance scheduling plan given the information that future funding is uncertain. The author also proposes an augmented Lagrangian decomposition method and a scenario reduction method to solve the stochastic programming problem. The usefulness and efficiency of the proposed model will be tested in a real road network maintenance scheduling problem.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1. INTRODUCTION

1.1 INFRASTRUCTURE ASSET MANAGEMENT

Infrastructure asset management (IAM) is a systematic approach of maintaining, upgrading, and operating infrastructure facilities cost effectively. Examples of infrastructure assets include pavements, bridges, drainage culverts and storm drainage systems, traffic signals, traffic signs, traffic striping, ITS infrastructure, safety rest areas and roadside. IAM combines engineering principles with sound business practices and economic theory, and it provides tools to facilitate a more organized, logical approach to decision making. The goal of infrastructure asset management is the effective management of large and complex infrastructure systems in an integrated manner, by considering the interdependency between all of the facilities within the system. As such, infrastructure asset management aims to provide information to decision makers about the trade-offs of different alternative solutions. Generally, the management process focuses on the stages of a facility's life cycle specifically maintenance, rehabilitation, and replacement. Asset management specifically uses mathematical models and computer software to organize and implement with the fundamental goal to preserve and extend the service life of long-term infrastructure assets which are vital underlying components in maintaining the quality of life in society and efficiency in the economy. In the broadest sense, infrastructure management covers all phases of infrastructure planning, design, construction, maintenance and disposal (Figure 0-1).

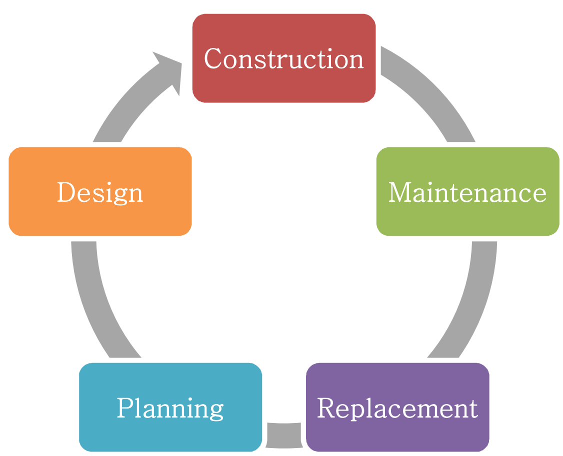

Figure 0-1 Life Cycle Phases of Infrastructure Asset Management

All infrastructure facilities deteriorate over time due to different reasons, including material, usage and environmental damage. Deterioration of infrastructure systems overtime is inevitable because of wear and tear caused by usage and that the materials that make up the facility begin to break down and become affected by elements such as rain, sunlight, and chemicals that come into contact with the surface. For example, the asphalt binder that is the "glue" of the pavement begins to lose its natural resistance to water, allowing moisture to penetrate into and underneath the pavement. The truth is, no infrastructure facility is exempt from deterioration no matter how well it is constructed. Material deterioration begins immediately. Even in normal conditions substantial deterioration can begin to take place after a certain period of time.

As infrastructure facilities deteriorate, the cost to operate and maintain them increases. Therefore, managing maintenance activities for large scale infrastructure systems is a difficult task. Many projects and interests compete for the limited resources allocated to different programs. Many factors are involved in the decision making process of infrastructure asset management. The basic elements of infrastructure asset management are shown in Figure 0-2.

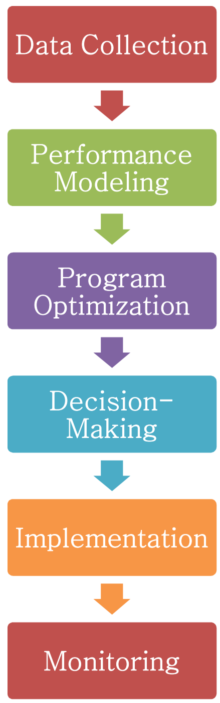

Figure 0-2 Basic Elements of Infrastructure Asset Management

1.1.1 Data Collection

This element of the management provides the decision makers with information on condition of the system for which the decision maker is responsible for managing. The system is usually divided into management facilities or segments. The data collected will provide basic information about the location and inter-connectivity of each management segment. The minimum data required for each management segment generally includes: identification, location, size, importance such as functional classification, material type, usage levels and date of construction or last major repair.

For example, for a network of pavements, the data need to be collected include distress, rutting, roughness, slab faulting, pavement strength, and so on. These indicators will be used in preparing the needs analysis/construction program, aid in pavement design and management and highway improvement. Also, physical information from highway construction and maintenance projects can also be inventoried.

1.1.2 Performance Modeling

Accurate prediction of infrastructure performance is critical to infrastructure asset management agencies. Reliable and accurate predictions of infrastructure performance can save significant amounts of money for infrastructure management agencies through better planning, maintenance, and rehabilitation activities. Infrastructure deterioration is a complicated, dynamic, and stochastic process affected by various factors such as design, climate conditions (e.g., rainfall, temperature, and amount of sunlight), material, structural capacities, as well as some unobserved factors. In general, the deterioration process of an infrastructure facility is a function of various factors affecting the mechanistic characteristics of the facility, such as design, climate, materials, construction, age, and the degree of maintenance. Deterioration models can be developed by using historical data as discussed in previous section. Usually, data points affected by maintenance activities are excluded when used in the development of deterioration models, in order to obtain the true deterioration process of the facility.

1.1.3 Program Optimization

After the performance models have been developed, the condition of individual facilities can be projected into the future. However, the projected condition may not satisfy the decision makers' requirement. Therefore, maintenance plans need to be adjusted and facilities will be selected for maintenance and repair during the planning horizon in order to achieve an established goal. Once an agency determines the funding needed to maintain the system in desired condition, the identified funding requirements will be compared to the funding available. Life-cycle cost analysis is usually adopted to compare different strategies. For an existing facility, the life-cycle cost analysis is considered an efficient approach for comparing the long-term impacts of different maintenance strategies and identifying the optimal ones. Using the life-cycle cost approach allows the decision maker to compare different strategies from the economic perspective and determine the most cost-effective one over a certain planning horizon. If available funding are less than those identified as needed for any of the years in the analysis period, optimization technique or other mechanism can be used to allocate the available funding among the management facilities, the goal of which is to provide the greatest overall return in system-level performance for the funding expected.

1.1.4 Feedback

Feedback refers to the transfer of part of the output to the input. A feedback system ensures continual feedback of information for assessing infrastructure system conditions. System conditions can be predicted using prediction models, and comparison with the feedback condition data provides a measure of predictive capabilities. It also provides procedures for evaluating other aspects of the facility network, including observed life cycle costs and performance of maintenance and rehabilitation treatments. Identification of dangerous spots is another area where feedback analysis can be extremely useful. A desired Infrastructure Management Decision Support Systems should be implemented using projection techniques, assignment processes, and costs based on limited information. For a system to become fully adopted and used, it must provide reliable projections. The feedback process therefore provides information to evaluate how reliable past estimates have been and provides a method to adjust future estimates. These feedback processes are necessary for decision makers to operate the infrastructure management system, update projection algorithms, assignment processes, and costs on a repeating basis. In summary, a feedback system provides for measurement and evaluation of performance of the system in service.

In the context of a pavement management system, the monitoring information is brought to make some comparisons like comparisons of actual costs of maintenance, rehabilitation, and reconstruction with those obtained in the pavement management system analysis; evaluations of field observations of pavement conditions with those predicted by pavement management system models.

1.2 Infrastructure Asset Management Maintenance Scheduling Problem

An efficient transportation infrastructure network is vital to economic and social development. Infrastructure maintenance consumes a significant proportion of the surface transportation budget, while the costs borne by the road-using public for vehicle operation and depreciation are even greater. Facilities must be designed and constructed with budget and other applicable constraints. Infrastructure maintenance management is one of the most important components of infrastructure management. It is the process of developing alternative maintenance strategies and determining the best solution to ensure desired level of service. For many types of infrastructure facilities, the service life can be extended beyond the original design life by applying maintenance treatments. Maintenance strategies are generally considered a sequence of treatments selected from a list of possible treatments generally available for the facility. Life-cycle cost concepts should be used to determine the difference in costs between various strategies. Costs should consider those borne by both user and agency.

Maintenance options of an infrastructure facility consist of various routine, preventive, or reactive activities, and other rehabilitation and replacement techniques. Maintenance expenditure is one of the costly infrastructure investments. From a mathematical point of view, there are two types of maintenance scheduling problems. The first one is network-level problem, where decision makers face great challenges of determining which facility is to be repaired, when and how repairs should be carried out, and what treatment to use. Another is the project-level maintenance problem, in which only the maintenance scheduling of one facility is considered.

1.2.1 Project Level Problem

Project level problem is the foundation of network-level problem. Project level problem is to determine the best strategy possible for a single facility within imposed constraints including available funds. The primary results of project-level management include an assessment of the cause of deterioration, identification of possible strategies, and selection of the "best" strategy given the constraints present. A typical project level maintenance scheduling problem is illustrated in Figure 0-3. As showing in the figure, there are two maintenance scheduling solutions (blue and red). It requires a life-cycle cost analysis before the decision makers can make a choice between them.

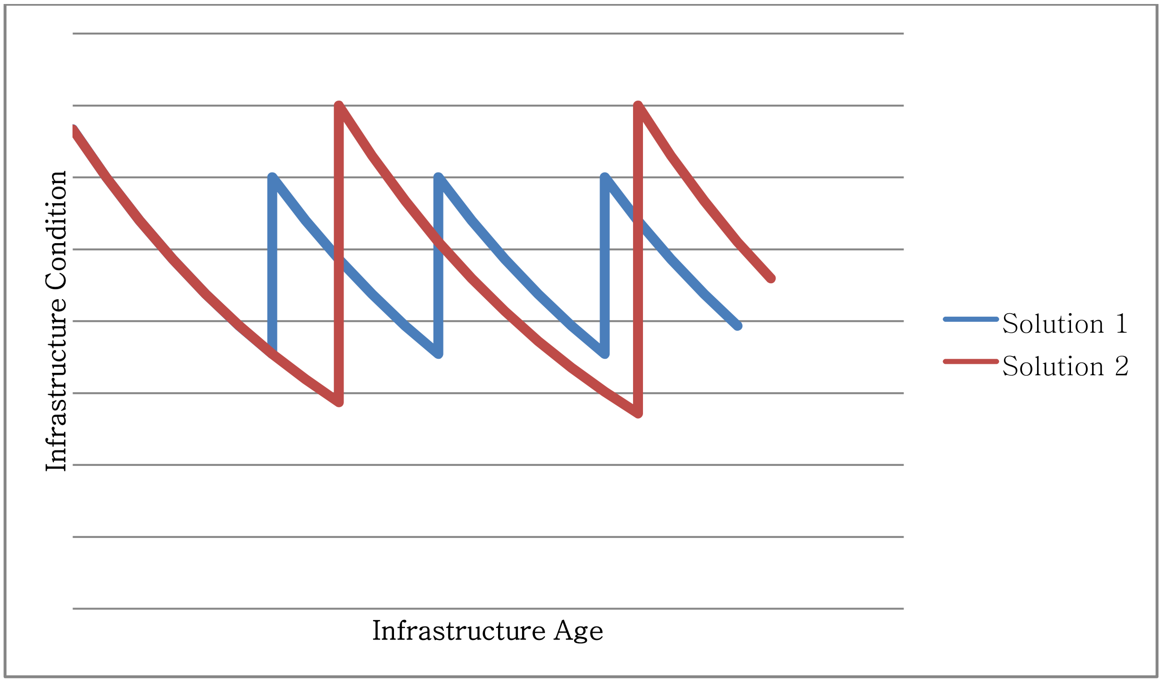

Figure 0-3 Project-Level Maintenance Scheduling Problem

1.2.2 Network Level Problem

Network Level problems deal with systems of multiple facilities. The model of network level problem is usually a combination of models of many project level problems. Therefore, network level problems are more complex and more difficult to solve than project level problems. In network-level maintenance scheduling problem, decision makers usually have to consider large-scale systems of facilities under their jurisdiction. The purpose of the network-level maintenance scheduling problem is normally to identify the fund needs and determine location and timing of maintenance treatments for the whole system.

1.3 MOTIVATION OF THIS RESEARCH

The U.S. population is expected to grow by 100 million and the number of miles traveled on the nation's highway will double during the next 30 years. However, the current investment levels are not keeping pace with the increased usage and deterioration of the highway network. Nearly 161,750 miles of federal-aid highways have pavement rated unacceptable and 153,990 bridges nationwide are structurally deficient or functionally obsolete, according to U.S. Department of Transportation (U.S. DOT) data.

This situation is not going to change in the next ten years unless steps are taken to improve how available funds are used and to increase the amount of funds to meet system needs. According to data from the 2006 U.S. DOT "Condition & Performance" report, the federal share of highway investment needed just to maintain highway conditions and performance will be $55 billion in FY 2010 and will grow to almost $62 billion by FY 2015. For those same years, the U.S. Treasury estimates that revenues into the Highway Account will start at $37 billion and grow to just under $42 billion. The gap between projected revenues and minimum investment needs average $19 billion per year. Under this situation, how to effectively use the limited funding on transportation infrastructure is a cutting-edge problem. A recent study conducted by Texas Department of Transportation shows that as a result of use and age, Texas' highway infrastructure is showing signs of deterioration. According to Federal Highway Administration data, passenger vehicle traffic in the United States is expected to increase by more than 30 percent by 2020, with large truck traffic estimated to increase by almost 40 percent. As indicated by the Texas Department of Transportation, a fully loaded tractortrailer truck damages the highway almost 10,000 times more than a passenger vehicle. Vehicle roadway damage affects smoothness of ride and causes ruts, potholes and cracks in the roadway. Driving on roads that are in disrepair accelerates vehicle deterioration, drives up roadway maintenance costs and increases fuel consumption. The total revenue available in Texas for pavement and bridge maintenance plus additional capacity is expected to be $100 billion from 2011 to 2035. The estimated funding gaps will range from $74 billion to $170 billion from 2011 to 2035 (Texas 2030 Committee, 2011).

A number of mathematical models have been developed for infrastructure system maintenance planning. Most of the approaches treat the annual budget as a fixed amount. An underlying assumption is that actual funds to support the maintenance activities would never deviate from the original expectation. However, this assumption is often unrealistic because the funding allocated to infrastructure maintenance program is subject to uncertainty due to various financial and political risks. Moreover, the funding for maintenance usually has to compete with other activities, e.g., capacity expansion projects. Consequently, the actual amount of money distributed to the maintenance activities may deviate from the original estimate. Therefore, if the funding falls short for some of the years during the planning period, part of the planned maintenance activities might be forced to be postponed, leading to inevitable condition deviation from the expectation. As a result, ignoring the random characteristics of future budget may limit the usefulness of the optimal scheduling solution. It is therefore without doubt that the assumption of deterministic budget is questionable in practice.

1.4 Objective

The objective of this research is to develop a network-level infrastructure maintenance scheduling problem under budget uncertainty. The problem was formulated as a multistage, linear stochastic programming model. The proposed model differs from its deterministic counterpart in that it attempts to find the optimal maintenance scheduling plan given the information that future funding is uncertain. The author also proposes an augmented Lagrangian decomposition method and a scenario reduction method to solve the stochastic programming problem. The usefulness and efficiency of the proposed model will be tested in a real road network maintenance scheduling problem.

1.5 Report Organization

The organization of this report is as follows: Chapter 1: Introduction. Chapter 2 focuses on reviewing the literature of infrastructure maintenance planning models and performance models. In this chapter, previous works are classified into different categories based on the nature of these models. For each category, the advantages and disadvantages are discussed and summarized. Chapter 3 describes the methodologies of formulating the infrastructure maintenance planning problem in both deterministic and stochastic settings. Chapter 4 discusses the solution methods used in this research to solve the multi-stage stochastic programming problem. Chapter 5 presents the application of the model and algorithm developed in Chapter 3 and 4 to the PMIS data set. The optimal solution results are discussed. Chapter 6 summarizes the research effort and presents the conclusions.

CHAPTER 2. LITERATURE REVIEW OF INFRASTRUCTURE PERFORMANCE MODELS AND MAINTENANCE SCHEDULING MODELS

2.1 Performance Models

In infrastructure asset management, performance models are used to predict future conditions and to help schedule maintenance activities. The effectiveness of maintenance planning in infrastructure management depends on the accuracy of the predicted future condition of infrastructure facilities. If the performance models used in determining the maintenance policies cannot effectively represent the actual deterioration process, the planned maintenance activities might not yield the expected results, which leads to suboptimal use of resources.

In general, the deterioration process of an infrastructure facility is a function of various factors affecting the mechanistic characteristics of the facility, such as design, environment, materials, construction, age, and the degree of maintenance. The deterioration process also involves load and load application sequence, and other factors that cause system responses such as stress, strain and deflections.

Performance models can be classified into two types: deterministic or probabilistic. In deterministic models, the future condition of a facility is predicted as an exact value based the past information collected about the facility. In probabilistic models, the performance of a facility is predicted by estimating the probability with which the facility would change to a particular condition state, from a predefined set of possible facility conditions of the random process. Probabilistic models are usually associated with discretization of the condition states. Moreover, probabilistic models can also be used to describe the deterioration of the whole system.

Most of the performance models developed in the early stages of infrastructure management research are deterministic (see AASHO (1962), Garcia and Riggins (1984), Paterson (1987), for example). Such models are unable to effectively take into consideration the stochastic nature of infrastructure deterioration. Infrastructure deterioration is a complex process that is associated with uncertainties. The uncertainties of infrastructure deterioration come from three sources. The first source concerns measurement errors, which can cause a high degree of prediction uncertainty (Humplick, 1992). The second source of uncertainty is the inherent randomness of the facility deterioration processes. The third source is the inability to model the true deterioration process, because the facility performance is also affected by other latent factors (e.g., construction quality), which are difficult to observe and quantify individually. Therefore, probabilistic models are developed to help take uncertainty into consideration when modeling infrastructure deterioration.

A popular example of probabilistic performance models is the one based on the Markov Chain, in which the deterioration process is characterized by transition between different condition states. Markov Chain can be used in modeling both single facility (e.g., pavement, bridge) and systems (e.g., pavement network). For example, Golabi et al. (1982) proved the effectiveness of using the Markov Chain method by developing Markov Chain performance models in Arizona. The core of the Markov Chain models is the development of the transition probabilities. A number of methods including the expected-value method by Butt et al. (1987) and Jiang et al. (1989) and the proportion method by Wang et al. (1994) have been employed to develop the transition probabilities. Another way of developing the transition probabilities is the simulation approach of utilizing the design equations (Gao et al. 2007). Other similar models were developed to consider the impact of some relevant factors. For example, Madanat et al. (1995) used the ordered probit technique to link transition probabilities to relevant explanatory variables. Madanat et al. (1997) further extend the probit model to account for the heterogeneity in the dataset. These models predict the facility conditions at fixed time points.

Another type of probabilistic performance model is the reliability model (or survival model). For example, Mishalani and Madanat (2002) developed a probabilistic model to determine the probability distribution of the time it takes an infrastructure facility to leave a condition state once entered. Mauch and Madanat (2001) conducted a similar study by developing the duration model from a semiparametric approach. Prozzi and Madanat (2000) developed a duration model to predict the number of axle load repetitions needed to reduce serviceability below an acceptable level. Zhang and Damnjanovic (2006) developed a model to predict the reliability of the pavement by using design equations. The limitation of reliability model is that the condition of an infrastructure facility (e.g., a pavement section) is usually characterized by multiple condition states. As a result, using only two states (survival and failure) cannot fully characterize the changing of the facility condition. The reliability-based model is more suitable for modeling a specific distress failure mode, in which the development of the distress cannot be easily observed until it reaches a certain level (see Wang et al. (2005), for example). It can also be used in such scenarios that the failure of a facility (e.g., a bridge) has significant consequence. In such case, decision makers can better understand the risk by using reliability models.

In the rest of this section, major existing performance models in the literature are discussed.

2.1.1 Markov Chain Model

The Markov Chain model used in infrastructure deterioration modeling is a stochastic process and is characterized by the following features. First of all, the Markov process is discrete in time. Second, the Markov process has a countable state space. Finally, the Markov process satisfies the Markovian property. The Markovian property is said to be satisfied if the future state of the process depends on its present state, but not on its past states. Therefore, for the prediction of infrastructure deterioration, this property is satisfied if the future condition of the facility is dependent on its present condition and not on its past condition. The Markov Chain model can be used for both project-level and network-level performance modeling (Gao et al. 2007). For project level problems, the condition probability represents the probability of the facility being in a specific state. For network level problems, the condition probability represents the proportion of network being in a specific state. The reason why Markov Chain model can be used in the determination of infrastructure facility deterioration is as follows. In general, infrastructure deterioration is a continuous process. However, the inspection of the condition is usually carried out at specific points in time, e.g., annually. The state space, that is the number of possible outcomes of the condition indicator, is infinite. However, especially at the management level, the state space is usually defined as a finite number of discrete condition states.

There are two types of discrete time Markov Chain models: stationary in time if the probability of going from one state to another is independent of the time; or non-stationary if the transition probability changes with the time. Infrastructure facilities can be modeled as either stationary or non-stationary. To apply the Markov Chain Theory (Figure 0-1), the condition of a facility is first discretized into n states. Hence, facility condition at different time periods can be represented by a condition state probability vector:

$$C(t) = [c_1(t), c_2(t), \dots \dots, c_n(t)] \tag{2.1}$$

where:
$C(t)$ = condition state probability vector of a facility at time period t;
$c_i(t)$ = probability that a facility stays in state i at time period t, $i = 1,2, \dots, n$ and $\sum_{i=1}^{n} c_i(t) = 1$.

Figure 0-1 Markov Chain Performance Model

The deterioration process of an infrastructure facility can be expressed by the change of the elements of the condition state probability vectors. A transition probability matrix P is used to simulate this change. In the Markovian process, it is assumed that the future condition states of a facility depend only on its current condition state. Any experience before it has no impact on the future condition. Therefore, in order to calculate future condition state probability, only the present condition state probability vector and P are needed.

$$P = \begin{pmatrix} p_{11} & p_{12} & 0 & \cdots & & 0 & 0 \\ 0 & p_{22} & p_{23} & \cdots & & 0 & 0 \\ 0 & 0 & p_{33} & \cdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & p_{n-1,n-1} & & p_{n-1,n} \\ 0 & 0 & 0 & \cdots & 0 & & p_{n,n} = 1 \end{pmatrix} \tag{2.2}$$

where,
p_{ij} = probability that the facility will deteriorate from state i to state j at time period t;
Because a facility cannot improve to a better condition state by itself, the elements p_{ij} are replaced by 0 for $i > j$. Furthermore, the value of 1 in the last row of the matrix corresponding to state n indicates that the condition cannot deteriorate further. From all the above, the future condition probability can be calculated by:

$$C(t+\Delta t) = C(t) P^{\Delta t} \tag{2.3}$$

2.1.2 Reliability Model (or Survival Model)

Survival analysis is a branch of statistics which deals with the counting of deaths and failures. More generally, survival analysis involves the modeling of time to event. Death or failure is considered an "event" in the survival analysis literature. Survival analysis attempts to answer questions of what is the probability of a subject surviving past a certain time; and what are the effects that affect the failure. The primary goal in using survival models to analyze infrastructure condition data is to assess the dependence of time-to-failure on external variables. One way to explore the relationship of covariates on time-to-failure is by means of a regression model in which failure time has a probability distribution that depends on the covariates.

The specific feature that distinguishes survival analysis from classical statistical analysis is data censoring. Usually, the failure time is unknown for some of the facilities. The only information available is that the facility has survived up to a certain time. Therefore, the facility is no longer followed up. This type of censoring is called right censoring. For right-censored data, the actual information of the ith facility $i = 1,...,n$ is contained in the pair (t_i, d_i), where t_i is the failure time and d_i is the censoring indicator, taking the value one if the event has been observed (failed), otherwise d_i takes value zero (censored). Then the censoring indicator can be expressed in Eq.(2.4).

$$d_i = \begin{cases} 1 & \text{if } t_i \le c_i \\ 0 & \text{if } t_i > c_i \end{cases} \tag{2.4}$$

where, c_i is the censoring time.

For a random time-to-failure, T, the probability density function of T is defined as $f(t)$ and the cumulative distribution function as $F(t) = P(T \le t)$. Two other functions that are useful in this context are the survival function $S(t) = P(T > t) = 1 - F(t)$, and the hazard function $h(t) = f(t)/S(t)$, which can be interpreted as the instantaneous rate of failure given survival up until time t.

One of the survival models that have been used in infrastructure performance modeling is the Proportional Hazard (PH) model (Gao et al. 2011). In general, a PH model with covariates can be written as:

$$h(t_i) = h_0(t_i)\exp(\mathbf{x}_i^{'}\boldsymbol{\beta}) \tag{2.5}$$

where $h_0(t)$ is the baseline hazard function representing the deterioration rate of the facility; $\boldsymbol{\beta}$ is the parameter vector and $\mathbf{x}_i$ is the covariates vector of the ith observation. A typical survival curve of an infrastructure facility is shown in Figure 0-2.

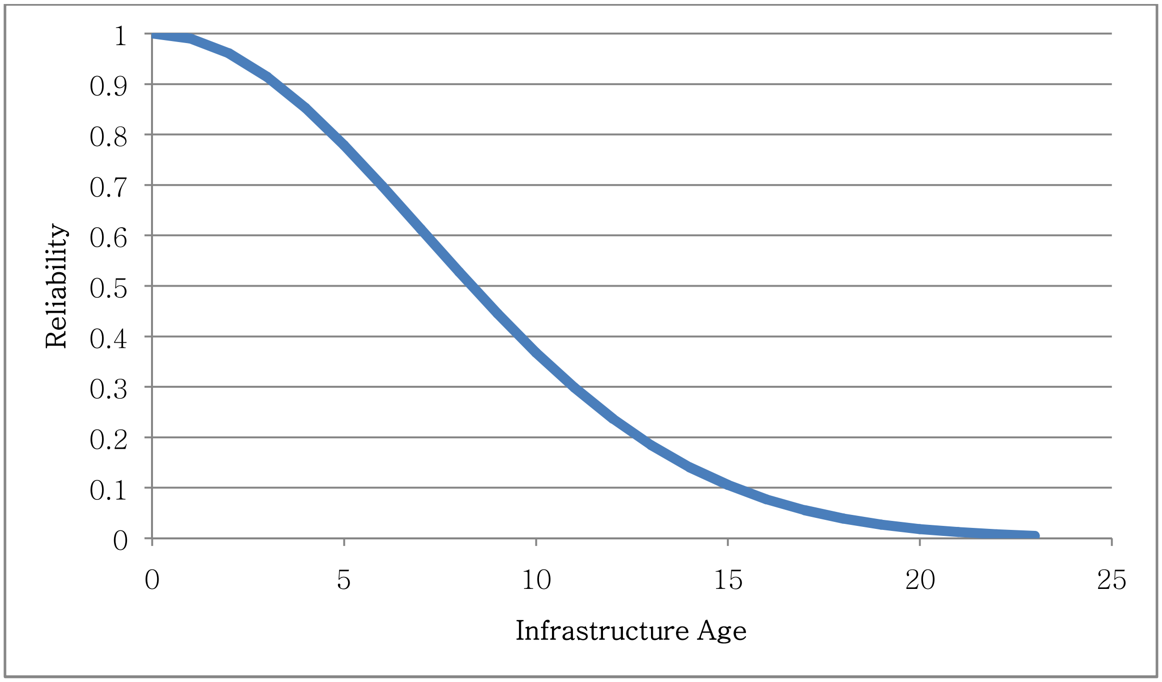

Figure 0-2 Survival Curve of an Infrastructure Facility

2.1.3 Discrete Choice Model

Another probabilistic model, discrete choice model, has also been used in the infrastructure performance modeling (see Zhang and Gao (2010), for example). Discrete choice problems involve choices between two or more discrete alternatives, such as staying or not staying in the good condition state. Discrete Choice Modeling is a powerful analytic technique for understanding the choice between alternatives. The modeling technique reveals the relationship between the probability of choosing an alternative and the attributes or benefits that characterize that alternative. More specifically, the discrete choice model is a mathematical representation of the preferences that provides estimates of the utility or value that the subject places on different features or benefits when making constrained choices. Discrete choice model is similar to reliability model, but different in the number of condition states. Instead of only defining two condition states, discrete choice model allows the existence of multiple condition states. Therefore, it can better capture the deterioration process than reliability models.

One of the discrete choice model applications in infrastructure performance modeling can be explained as follows. Let C_n as the dependent variable represent the condition state for facility n and an underlying response variable U_n be a measure of the latent deterioration propensity for facility n. U_n is assumed as a continuous variable varying from $-\infty$ to $+\infty$. The observed facility condition state k is a reflection of the latent variable U_n, which is specified to be a summation of a deterministic function of explanatory variables. In this case, the structure of the model can be described as:

$$U_n = \beta' X_n + \varepsilon_n \ \ (n = 1,2,.,,,N) \tag{2.6}$$

where U_n is the underlying response variable; X_n is a set of explanatory variables; β is the estimated parameter; and ε_n is the error term. The above equation cannot be directly estimated, since U_n is not observable. But the observable state k that facility n falls in can be used to estimate the parameters in the model. As such, C_n is governed by Ψ_k, the threshold values of the underlying response variable U_n. If the latent variable falls between the threshold Φ_k and Φ_{k-1}, then the C_n falls into the corresponding state k. In this regard, the thresholds separate the continuous underlying response variable U_n into different states. If ε_n is assumed to follow a standard normal distribution with mean 0 and standard deviation 1. Then the probability for facility n to be in the condition state k can be obtained by

$$P(C_{nk} = 1) = \Phi(\Psi_k - \beta' X_n) - \Phi(\Psi_{k-1} - \beta' X_n) \tag{2.7}$$

2.1.4 Deterministic Models

Deterministic models are usually used for single facility performance modeling. Deterministic performance models can usually be expressed in a general form (2.8). Let $\mathbf{y} = (y_1, ..., y_n)$ be a $n \times 1$ vector of sample of n condition observations expressed as:

$$y_i = h(\mathbf{x}_i, \boldsymbol{\beta}) + \varepsilon_i \tag{2.8}$$

where h is the deterioration function. $\mathbf{x}_i$ is a $1 \times p$ vector of p explanatory variables and $\boldsymbol{\beta}$ a $p \times 1$ vector of the corresponding coefficients. The error term ε_i is assumed to follow a certain distribution with associated coefficients $\boldsymbol{\theta}$. A data point affected by maintenance intervention can be modeled as:

$$y_i = h(\mathbf{x}_i, \boldsymbol{\beta}) + \delta_i A_i + \varepsilon_i \tag{2.9}$$

where δ_i is an independent Bernoulli trial with success probability τ representing the existence of the maintenance intervention. A_i represents maintenance effectiveness and is assumed to follow a probability distribution g with parameters $\boldsymbol{\kappa}$. Based on (2.9), the deterioration rate is captured by estimating the parameter $\boldsymbol{\beta}$. Moreover, by estimating the parameters δ_i and A_i, the model determines if the i^{th} observation is affected by maintenance treatment and if so, the magnitude of the impact. Details of the model can be found in Gao et al. (2011) and Hong and Prozzi (2010).

2.2 Maintenance Scheduling Models

Maintenance scheduling models include the mathematical models focused on finding either the optimal balance between costs and benefits of maintenance or the most appropriate time to execute maintenance. Parameters often considered in this optimization are the cost of failure, the cost per time unit of downtime, the cost (per time unit) of corrective and preventive maintenance and the cost of repairable system replacement. The foundation of any maintenance scheduling model relies on the underlying deterioration process and failure behavior of the facility. Maintenance scheduling optimization is one of the most critical issues in infrastructure asset management since the failure of a system during actual operation can be a costly and dangerous event. When a facility fails to operate in a system, it does not only cause damage to the system but also affect all the users. This optimization process can utilize different methods. It can be made by adding features and conditions that make the maintenance policy more realistic.

Numerous efforts have been made to develop mathematical models as maintenance strategy decision-making aids. The infrastructure maintenance management problem can generally be formulated in both discrete-time and continuous-time settings. In discrete-time setting models, a set of time points at which the maintenance treatment might be applied is predefined, for example, at the beginning or end of each year. The solution of this type of model determines which maintenance treatment should be applied at specific time points. In practice, infrastructure agencies make maintenance decisions subject to budgetary constraints and resource availability. Agencies allocate resources for maintenance activities at the beginning of each budgeting year. It is therefore realistic to discretize the planning horizon into predetermined temporal stages (e.g., years) and restrict treatments to occur only at such time points. In continuous-time setting models, however, there is no predefined constraint about the timing of the maintenance treatment. The solution of this type of model determines both timing and type of maintenance treatments.

In the rest of this section, some popular maintenance scheduling models are discussed.

2.2.1 Ranking Method

A simple ranking procedure can be used for network-level maintenance scheduling. It generally ranks those in the worst condition as the highest priority without regard to the return on the funds expended. If the goal is to provide the best service for the available funds, then some type of measure of cost-effectiveness should be included in the selection process. The advantage of this method is its easy-to-use feature. However, the resulting funding allocation is not optimal. Better allocation scheme can be found by using other methods.

2.2.2 Markov Chain Based Linear Programming (LP)

The Markov Chain based linear programming model is a discrete-time setting model. It is usually used for network-level infrastructure maintenance scheduling problem. In the LP model, facilities with similar deterioration patterns are grouped together. The solution of this model determines the percentage of a group's maintenance strategy instead of the strategy for each facility. Therefore, the computational effort of this model type is simpler than the Integer Programming models (see 2.2.3) (Smilowitz and Madanat 2000; Guignier and Madanat 1999; Robelin and Madanat 2006; Wu et al. 2009; Gao et al. 2010). However, since the solution only determines the portion of the system that will receive a certain maintenance treatment, it requires additional effort to determine the specific facilities. For example, if the solution indicates that 20% of a road network should receive preventive maintenance treatments, it needs some mechanism to determine which pavement section should be in that 20%. Nevertheless, different ways of determining that 20% will not change the objective function value in the LP model.

The mathematical expression of the LP model can be explained as follows. Consider an infrastructure system as a set $\mathcal{S} = \{1,2,\dots,S\}$ of different groups of facilities with homogeneous properties, e.g., by highway functional class. $\mathcal{I} = \{1,2,\dots,I\}$ is defined as a set of state space with elements representing the facility condition state. Each element of this set represents a specific condition state. In each time period, a decision should be made to determine the proportion of system that should receive maintenance treatment and the type of treatment that should be applied. A set of basic maintenance treatments is defined as $\mathcal{M} = \{1,2, \ldots, M\}$, where the Mth treatment is set to be most effective and also expensive. The scheduling time horizon is represented by the discrete set of time periods $\mathcal{T} = \{1,2, \ldots, T\}$. During each time period, the conditions of facilities deteriorate because of usage, aging, and environment. The deterioration process of a pavement can be expressed by the change of the elements of the condition state probability vectors. More details of the Markov Chain based linear programming model will be discussed in Chapter 3.

Linear programming can be efficiently solved by the simplex algorithm, which solves Linear Programming problems by constructing a feasible solution at a vertex of the polytope and then walking along a path on the edges of the polytope to vertices with non-decreasing values of the objective function until an optimum is reached. In general, the simplex algorithm is very efficient and can be guaranteed to find the global optimum if certain precautions against cycling are taken. Multiple optimal solution is also possible in Linear programming problems.

2.2.3 Integer Programming (IP)

The IP model is another discrete-time setting approach to solve multiple facilities and budget constraint problem. The advantage of IP over LP is that its solution will assign maintenance treatment directly to individual facilities. However, it is usually used on small size of systems because the computational burden of combinatorics. Wang et al. (2003) developed a multi-objective IP model for network-level pavement maintenance management. In this paper, the authors use the branch and bound algorithm to solve the proposed model. Ouyang and Madanat (2004) also developed an IP model outlining the scheduling of rehabilitation activities for multiple pavement facilities. They proposed a greedy heuristic to solve the problem. However, due to the combinatory nature of the IP approach, the computational burden of network-level maintenance management problems increases exponentially as the number of facilities under consideration increases. Therefore, some researchers tend to use approximation techniques when dealing with large-scale facility maintenance scheduling. For example, Gao and Zhang (2008) use the approximate dynamic programming method to solve network-level pavement management problem. Karabakal et al. (1994) and Dahl et al. (2008) use Lagrangian relaxation techniques to decompose the network-level IP problem into simpler project-level dynamic programming problems.

The mathematical formulation of the IP approach can be explained as follows. Let $\mathcal{T} = \{1,2, \ldots, T\}$ represent the set of planning horizon. $\mathcal{A}$ is defined as a set with N elements representing facilities in the system. A set of basic maintenance treatments is defined as $\mathcal{M} = \{1,2, \ldots, M\}$, where the Mth treatment is set to be the most effective and expensive. Given the initial condition of facility a, s_a^0, and the deterioration function $f(\cdot)$, the IP formulation is:

$$\max \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} s_a^t \tag{2.10}$$

$$\text{s.t.} \sum_{a \in \mathcal{A}} \sum_{m \in \mathcal{M}} c_{amt} u_{amt} \leq B_t, \forall t \in \tag{2.11}$$

$$s_a^t = f(s_a^{t-1}) + \sum_{m \in \mathcal{M}} u_{amt} e_m, \quad \forall a \in \mathcal{A}, t \in \mathcal{T} \tag{2.12}$$

$$\sum_{m \in \mathcal{M}} u_{atm} \leq 1, \quad \forall a \in \mathcal{A}, t \in \mathcal{T} \tag{2.13}$$

$$u_{atm} \in \{0,1\}, \forall a \in \mathcal{A}, t \in \mathcal{T}, m \in \mathcal{M} \tag{2.14}$$

$$s_a^t > 0, \forall a \in \mathcal{A}, t \in \mathcal{T} \tag{2.15}$$

where,
c_{amt} = maintenance cost of applying the mth treatment to facility a at year t;
B_t = budget at year t;
u_{amt} = binary variable, equals to 1 if the mth treatment is applied to facility a and equals to 0 otherwise;
s_a^t = condition of facility a at year t;
$f(\cdot)$ = deterioration function;
e_m = maintenance effectiveness of the mth treatment.

The objective function (2.10) is to maximize the sum of every year's condition of all facilities. Constraint (2.11) states that the annual expenditure cannot exceed the available budget. Constraint (2.12) represents the deterioration process of the infrastructure facility. Constraint (2.13) states that only one treatment can be applied to the same facility each year. Constraints (2.14) and (2.15) define the decision variables of the IP model.

Methods of solving MIP problems can be largely classified into the following categories:

1. Branch and Bound method. It is the most widely used method for solving MIP problems. Subproblems are created by adding constraints to the integer variables. For example, if the kth integer variable x_k's current solution is u, which is not an integer. Then the original problem is divided into two problems with respect to x_k, with $x_k \leq \lfloor u \rfloor$ and $x_k \geq \lceil u \rceil$ respectively. Lower bounds are obtained by the linear-programming relaxation to the problem. It is implemented by keeping the objective function and all constraints, but relaxing the integrality constraints. If the optimal solution to a relaxed problem is integral, it is an optimal solution to the subproblem, and the value can be used to terminate searches of subproblems whose lower bound is higher.

2. Branch and Cut method. Brach-and-cut methods are exact algorithms for integer programming problems. It solves the integer programming problem by using a combination of cutting plane method with a branch-and-bound algorithm. It works by solving a sequence of linear programming relaxations of the integer programming problem. Cutting plane methods improve the relaxation of the problem to more closely approximate the integer programming problem, and the branch-and-bound algorithms proceed by a sophisticated divide and conquer approach to solve problems. Branch-and-cut algorithm:

   Step 1. Initialization: Denote the original integer programming problem (2.1)-(2.8) as the root node and store it in the waiting node list $List$. Set the upper bound to be $UB :=$

$+\infty$ {best found}, the lower bound to be $LB := -\infty$ {Best Possible} and current best solution $\mathbf{x}^* := \emptyset$. Go to step 2.

Step 2. Termination: If $List = \emptyset$, then the current best solution x^* which yielded the objective value obj is optimal; if $\mathbf{x}^* = \emptyset$, then the original problem is infeasible. If $List \neq \emptyset$, go to step 3.

Step 3. Node selection: Select and delete a node from $List$. Go to step 4.

Step 4. Relaxation: Solve the linear programming relaxation of the selected node problem. If the relaxation is infeasible, node is deleted. If an optimal integer solution $\mathbf{x}_r$ is found and $obj < UB$, set $UB := obj$, $\mathbf{x}^* := \mathbf{x}_r$, remove nodes j from $List$ with $LB_j > UB$ and go to step 2. If an optimal integer solution $\mathbf{x}_r$ is found and $obj \geq UB$, remove nodes j from $List$ with $LB_J > UB$ and go to step 2. If the optimal solution $\mathbf{x}_r$ is not integer, go to step 5.

Step 5. Add cutting planes: search for cutting planes that are violated by $\mathbf{x}_r$; if any are found, add them to the relaxation and go to step 4. If no cutting planes are found, go to step 6.

Step 6. Branching: find variable x_k of solution $\mathbf{x}_r$ with fractional value v. Create node j_{new} with bound $x_k \leq \lfloor v \rfloor$ and set $LB_{j_{new}} := obj$. Store node j_{new} in $List$. Create node j_{new} with bound $x_k \geq \lceil v \rceil$ and set $LB_{j_{new}} := obj$. Store node j_{new} in $List$. Go to step 2.

3. Branch and Price method. This is essentially branch and bound combined with column generation. This method is used to solve integer programs where there are too many variables to represent the problem explicitly. Thus only the active set of variables is maintained and columns are generated as needed during the solution of the linear program.

2.2.4 Reliability Model

Reliability model is one of the continuous-time setting models. It is usually used to model maintenance plan for infrastructure facility whose failure has significant consequence (e.g., bridge, traffic lights). An infrastructure facility operates with excellent efficiency when it is new, but as it ages its performance deteriorates. According to the model types, for single facility, reliability models can be classified into several subcategories. These may include age replacement models, minimal repair models, and inspection/maintenance models, Age replacement models deal with optimal replacement policies, which are based on age dependent operating costs. Minimal repair models focus on repairing a failed unit rather than from replacing it. They usually combine a periodic replacement policy with a minimal repair activity upon a unit failure. Finally, inspection/maintenance models are concerned with maintenance policies in which the current state of a system is not known but is available through an inspection. For multi-facility systems, reliability maintenance models concerns with optimal maintenance policies for a system consisting of several units of facilities, which may or may not depend on each other. Multi-facility reliability maintenance models can be divided into block or group maintenance models, inventory models and opportunistic models.

One example of reliability model is when the decision makers wish to minimize an optimal replacement policy that will minimize the sum of operating and replacement costs per unit time. The replacement policy is to perform replacements at intervals of length t_r. The objective is to determine the optimal interval between replacements to minimize the total cost of operation and replacement per unit time. The total cost per unit time, for replacement at time t_r, can be expressed as

$$C(t_r) = c(t) + C_r = \frac{1}{t_r}\left[\int_0^{t_r} c(t)dt + C_r\right] \qquad (2.16)$$

where,
$c(t)$ = operating cost per unit time at time t after replacement;
C_r = cost of a replacement.

2.2.5 Optimal Control Model

The optimal control model is another continuous-time setting approach to model project level maintenance scheduling problems. Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. A control problem includes a cost functional that is a function of state and control variables. An optimal control is a set of differential equations describing the paths of the control variables that minimize the cost functional. The optimal control can be derived using Pontryagin's maximum principle or by solving the Hamilton-Jacobi-Bellman equation.

The advantage of optimal control models is that maintenance actions are not restricted at fixed time points; the model calculates or determines the optimal maintenance time. However, unlike ordinary optimal control problems, the control actions (maintenance treatments) for infrastructure maintenance management are impulsive, leading to sudden jumps in the facility's condition. Therefore, special techniques are required to solve the problem. Previous researchers have adopted different approaches to address this issue. For example, Tsunokawa and Schofer (1994) developed an approximate method for road maintenance scheduling problems. In their paper, the impulse control problem is simplified by approximating discrete controls using ordinary continuous controls. The simplified problem can then be solved effectively using the Pontryagin's maximum principle. In another paper, Li and Madanat (2002) solved the same problem given the assumption that the planning horizon is infinite and the condition of the facility will enter a steady state after the first maintenance treatment. In this assumption, the optimal resurfacing strategy is to define a minimum serviceability level. When the facility condition deteriorates to that level, the strategy is employed to bring the condition back the best condition limit. In this manner, the authors obtained an optimal solution for the problem under steady state. Ouyang and Madanat (2006) derived an exact analytical solution for the same resurfacing problem with a finite planning horizon. Through variational derivation, the author obtained the necessary condition of the control problem.

As described by Tsunokawa and Schofer (1994), the condition of an infrastructure facility (e.g., International Roughness Index), denoted by s, usually follows a special trajectory curve over time as the facility deteriorates and receives maintenance treatments shown in Figure 0-3.

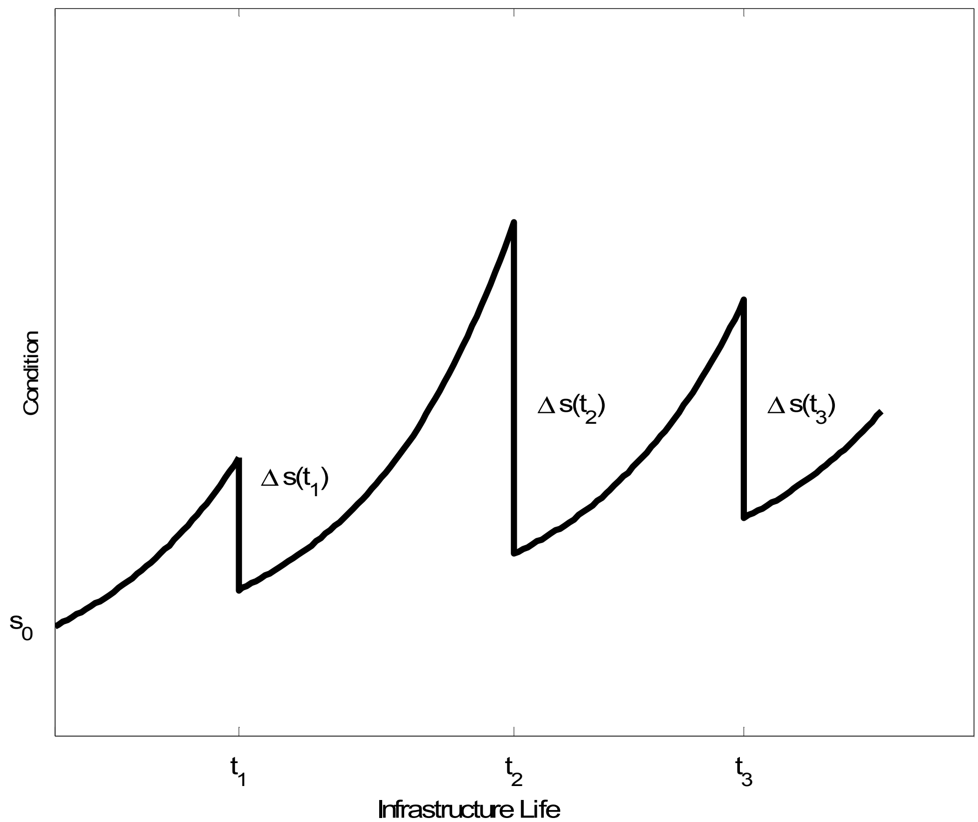

Figure 0-3 Infrastructure Condition Trend Trajectory

The deterioration rate of the facility is assumed to be a function, f, of the current condition level and expressed as follows:

$$\dot{s}(t) = f(s(t)) \tag{2.17}$$

The amount of condition improvement after a maintenance treatment is assumed to be a function, g, of the maintenance intensity (e.g., thickness of overlay), w, and the condition level immediately before the treatment, written as:

$$\Delta s(t) = g(s(t), w) \tag{2.18}$$

The initial condition is expressed as:

$$s(0) = s_0 \tag{2.19}$$

Costs for the agency and user are assumed to be functions of the condition and the maintenance intensity, respectively, and are written as $C(s)$ and $M(w)$. Using these functions, the total life-cycle costs for the agency and user of an infrastructure facility can be written as follows:

$$J = \int_0^T C(s(t))dt + \sum_{i=1}^{N} M(w(t_i)) \tag{2.20}$$

In this formula, T is the planning horizon, i represents the ith maintenance action, and N is the number of maintenance actions during the horizon. The problem of finding the optimal maintenance strategy for a given infrastructure facility can be defined as ascertaining the optimal values of t_i and $w(t_i)$, which minimize the life cycle cost, while the condition trajectory, $s(t)$, is determined by the Eqs. (2.17) and (2.18) and the initial condition (2.19). Therefore, the problem introduced above can be solved as an impulse control problem. The control problems described above arise frequently in applications. There are two basic approaches to their solution. One results in unconventional quasi-variational inequalities and uses the dynamic programming methodology. The other approach, similar to the classical calculus of variations, formulates the optimality necessary conditions in terms of maximum principle.

2.3 Infrastructure Maintenance Scheduling Models Considering Budget Uncertainty

The aforementioned models treat the annual budget as a fixed amount. An underlying assumption is that the actual funds to support maintenance activities never deviate from the original expectation. Under this assumption, the planning problem can be solved optimally using the optimization model mentioned previously. However, this assumption is often unrealistic, because the funding allocated to address infrastructure maintenance problems is usually subject to uncertainty due to various financial and political risks. Consequently, the actual amount of money distributed to maintenance activities may deviate from the original estimate. If funding falls short for some years during the planning period, some of the planned maintenance activities may be suspended, leading to inevitable condition fluctuation from the expectation. Therefore, ignoring the random characteristics of the future budget may limit the usefulness of the optimal planning solution.

In recent years, several researchers addressed the problems of budget uncertainty in the infrastructure management area. For example, Li and Puyan (2006) formulated a highway project selection problem under budget uncertainty as a multi-choice multidimensional Knapsack problem with multi-stage budget recourses. In their paper, the objective is to select a subset of candidate projects to achieve maximized system benefits under budget and other constraints. Gao and Zhang (2008) investigated the uncertainties in the pavement deterioration process and proposed a robust optimization approach for project-level maintenance planning problem. Using this approach, the decision maker is able to control the probability of achieving a certain level of condition requirement by adjusting the amount of money invested. Wu and Flintsch (2009) proposed a chance-constrained programming model with the ability to control the probability of going over budget for network-level facility maintenance planning problems. The solution of the proposed model is obtained by first choosing a conservative value for the budget and then treating the budget as fixed. However, the obtained scheduling solution of this model is only optimal at a given probability.

In this research, the network-level infrastructure maintenance scheduling problem under budget uncertainty is formulated as a multi-stage, linear stochastic programming model. Stochastic programming is a framework for modeling optimization problems that involve uncertainty. The goal of stochastic programming is to find a solution which is feasible for all data scenarios and optimal in some sense. Stochastic programming models take advantage of the fact that probability distributions governing the data are known or can be estimated. The proposed stochastic programming approach differs from its deterministic counterpart in that it attempt to achieve the best expected objective value over all possible realizations of the random parameters.

CHAPTER 3. PROBLEM FORMULATION

The mathematical formulation of the model developed in this research is presented in this section. The performance model is first discussed. Then the deterministic version of the infrastructure maintenance planning problem is presented. Finally, the author introduces the stochastic extension of the formulation by considering budget uncertainty.

3.1 NOTATIONS

Table 3.1 Notation

Sets	
$\mathbf{S}$	set of facility groups and $\mathbf{S}=\{1,2,...,S\}$
$\mathbf{I}$	set of facility condition states and $\mathbf{I}=\{1,2,...,I\}$ with I represents the worst condition state
$\mathbf{M}$	set of maintenance treatments and $\mathbf{M}=\{1,2,...,M\}$ with the M th treatment being the most effective and expensive
$\mathbf{T}$	set of planning periods $\mathbf{T}=\{1,2,...,T\}$
$\mathbf{K}$	set of all nodes in the scenario tree and $\mathbf{K}=\{1,...,K\}$, where $k=1$ corresponds to the root node at $t=1$ and $t(k)$ denotes the year corresponding to node k
$\mathbf{N}$	set of scenarios and $\mathbf{N}=\{1,2,...,N\}$
Parameters	
B_t	available budget at time period t
$\tilde{B}_t$	random variable representing available budget at time period t
B_t^n.	realization of the budget random variable $\tilde{B}_t$ in scenario n
b_t	number of realizations of $\tilde{B}_t$
C_{smt}	unit cost of applying the m th treatment to the s th facility group at time period t
L_s	number of the s th facility group
P_{sijm}	deterioration transition probability from condition state i to state j when the m th treatment is applied to the s th facility group. P_{sijm} satisfies the constraint of $\sum_{j\in\mathbf{I}} P_{sijm}=1, s\in\mathbf{S}, (i,j)\in\mathbf{I}, m\in\mathbf{M}$
X_{si1}	proportion of the s th facility group in condition state i at the beginning of the first time period, which is known to the decision maker before the maintenance planning
X^*	minimum requirement on the proportion of facilities in the first condition state

$P(t)$	condition state probability vector of a facility at time period t
$p_i(t)$	probability that a facility stays in state i in time period t; $i \in \mathbf{I}$, larger value of i corresponds to worse condition state; $\sum_{i=1}^{n} p_i(t) = 1$
D	transition probability matrix
d_{ij}	probability that the facility will deteriorate from state i to state j in one time period, if $i < j$ and $(i, j) \in \mathbf{I}$; probability that the facility will stay in the same state in one time period, if if $i = j$ and $(i, j) \in \mathbf{I}$
p^n	probability of occurrence of the nth scenario and $\sum_{n \in \mathbf{N}} p^n = 1$
Variables	
X_{simt}	proportion of the s th facility group in condition state i that receives the m th treatment at time period t
X_{simt}^{n}	decision variable X_{simt} for scenario n
X_{sit}	proportion of the s th type facility in condition state i at time period t
M_{smt}	proportion of the s facility group that receives the m th treatment at time period t

3.2 DETERIORATION MODELING

The concept of infrastructure condition is developed to quantitatively relate the condition of a facility to its ability to serve its intended users. Infrastructure condition is often represented by discrete ratings or states. Using discrete ratings instead of continuous indicators simplifies the computational complexity of the maintenance decision-making process, as details are not necessary at this level of management. Infrastructure facility condition deteriorates due to a variety of factors such as environmental conditions, and certain unobserved causes. In this paper, the deterioration of a facility is assumed to follow a discrete, state-based model widely used in infrastructure deterioration modeling (Wang et al. 1994; Li et al. 1996; Abaza et al. 2004; Gao and Zhang 2007).

The basic idea of the discrete, state-based model is introduced as follows. Facility condition at different years is represented by a condition state probability vector:

$$P(t) = [p_1(t), \cdots\cdots p_I(t)]^T \tag{3.1}$$

The deterioration process of a facility can be expressed by the change of the elements of the condition state probability vectors $P(t)$. A transition probability matrix D can be used to simulate this change.

$$D = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1I} \\ 0 & d_{22} & \cdots & d_{2I} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \tag{3.2}$$

Because a facility cannot improve to a better condition state by itself, the elements d_{ij} is replaced by 0 for $i > j$. Furthermore, the value of 1 in the last row of D corresponding to state I indicates that the condition cannot deteriorate further. From all the above, the future condition can be predicted as:

$$P(t+1) = D \times P(t) \tag{3.3}$$

where, $P(t+1)$ represents the condition state probability vector at time $t+1$;

3.3 Maintenance Planning Model with Deterministic Budgets

Consider an infrastructure system as a set $\mathbf{S}$ of facilities, e.g., pavements, bridges, rail, mass transit, and dams. Condition $\mathbf{I} = \{1, 2, ..., I\}$ is defined as a set of state space with elements representing the facility condition in which 1 represents the best condition state and I the worst. A set of basic maintenance treatments is defined as $\mathbf{M} = \{1, 2, ..., M\}$, where the M th maintenance treatment is set to be most effective and expensive. The scheduling time horizon is represented by the discrete set of time periods $\mathbf{T} = \{1, 2, ..., T\}$. During each time period, the conditions of the facilities deteriorate due to usage, aging, and environment. The effectiveness of maintenance treatment at time period t is reflected in the condition at time period $t+1$.

Using the discrete, state-based deterioration model, the infrastructure maintenance planning problem with deterministic budgets is formulated in Equations. (3.4)-(3.8).

$$\max \frac{1}{\sum_{s \in \mathbf{S}} L_s} \frac{1}{T+1} \left(\sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{m=1}^{M} L_s X_{s1mt} + \sum_{s=1}^{S} \sum_{i=1}^{I} \sum_{m=1}^{M} P_{i1m} L_s X_{simT} \right) \tag{3.4}$$

$$\text{s.t.} \sum_{m=1}^{M} X_{sim1} = X_{si1}, \forall s \in \mathbf{S}, i \in \mathbf{I} \tag{3.5}$$

$$\sum_{m=1}^{M} X_{sjmt} = \sum_{m=1}^{M} \sum_{i=1}^{I} P_{sijm} X_{sim,t-1}, \forall s \in \mathbf{S}, j \in \mathbf{I}, t = 2, ..., T \tag{3.6}$$

$$\sum_{s=1}^{S} \sum_{i=1}^{I} \sum_{m=1}^{M} C_{smt} X_{simt} L_s \le B_t, \forall t \in \mathbf{T} \tag{3.7}$$

$$0 \le X_{simt} \le 1, \forall s \in \mathbf{S}, i \in \mathbf{I}, m \in \mathbf{M}, t \in \mathbf{T} \tag{3.8}$$

The objective (3.4) of the planning problem is to maximize the proportion of all facilities in the best condition state over the planning horizon. The first term inside the parenthesis represents the proportion from time period 1 to time period T. The second term in the parenthesis represents the proportion at time period $T+1$, because a facility's condition at time period $T+1$ is fully determined by its condition and received maintenance treatments at time period T. Constraint (3.5) represents the initial condition of each facility group at the beginning of the planning horizon. Constraint (3.6) represents the deterioration process of the facilities between two consecutive time periods. Constraint (3.7) ensures that the annual expenditure of maintenance activities does not exceed the budget. Once the decision variables X_{simt} of problem (3.4)–(3.8) are obtained, the condition of each facility group can be calculated as:

$$X_{sit} = \sum_{m=1}^{M} X_{simt}, \forall s \in \mathbf{S}, i \in \mathbf{I}, t \in \mathbf{T} \tag{3.9}$$

The maintenance decision is then calculated as:

$$M_{smt} = \sum_{i=1}^{I} X_{simt}, \forall s \in \mathbf{S}, m \in \mathbf{M}, t \in \mathbf{T} \tag{3.10}$$

3.4 Maintenance Planning Model Under Budget Uncertainty

In this research, the budget uncertainty in the infrastructure maintenance planning problem is modeled using the Stochastic Programming model. Stochastic Programming is a framework for modelling optimization problems that involve uncertainty. In deterministic optimization problems, models are formulated with known parameters. However, in practice, many problems have some unknown parameters. Stochastic Programming model takes advantage of the fact that probability distributions governing the data are known or can be estimated. The objective is to find some solution that is feasible for all the possible data scenarios and maximizes (or minimize) the expectation of some function of the decisions and the random variables.

In a multi time period infrastructure maintenance planning problem, the budgets of time period 2 to T are unknown to decision makers at period 1. Therefore, to extend the deterministic formulation (3.4)-(3.8) to a stochastic setting, the budget B_t at time period t, $t = 1, \cdots\cdots, T$ is replaced with a random variable $\tilde{B}_t$. In this research, $\tilde{B}_t$ is assumed to evolve as a discrete time stochastic process with a finite probability space represented in the form of a scenario tree (for example Figure 0-1).

T stages in the tree represent T planning periods. The nodes at the t stage of the tree correspond to scenarios of possible values of $\tilde{B}_t$. If b_t represents the number of realizations of $\tilde{B}_t$, then there are $\prod_{t=1}^{T} b_t$ nodes at the Tth stage of the tree.

Furthermore, let $\mathbf{K} := \{1, ..., K\}$ denote the set of all nodes, where $k = 1$ corresponds to the root node at $t = 1$ and $t(k)$ denotes the time period corresponding to node k. Each node k is connected to its parent node k^+ at time period $t-1$ by an arc. A set of child nodes is associated with each node k with $t(k) \in \{1, ..., T-1\}$. The node set $\left(1, ..., \left(k^+\right)^+, k^+, k\right)$ is defined as a path from the root to node k.

A set $\mathbf{N} = \{1, 2, ..., N\}$ is defined as the scenarios with each element representing a path from the root to any nodes k with $t(k) = T$. A scenario represents one possible combination of values for all uncertain budgets. The probability associated with a scenario is the probability of reaching the corresponding node at year T from the root node. For each scenario, the associated probability is p^n and $\sum_{n \in \mathbf{N}} p^n = 1$.

To illustrate the concept of the scenario tree, a simple example is presented with a planning period of three years (Figure 3.1). The budget at the starting time period is already known to the decision maker. It is assumed that there are two possible values, \$5 million and \$7 million, for both the second and third year budgets. Therefore, four possible scenarios $(N = 4)$ may occur over the three decision periods. With the scenarios defined above, a probability of $p^n = 0.25, n \in \{1,2,3,4\}$ is assigned to each scenario. In an actual problem, the decision makers can assign any probability to each scenario based on their own judgement.

As illustrated in Figure 0-1, the scenario tree divides into branches corresponding to different realizations of the budget random variable. For example, the budget at year 2 is \$5 million for scenarios 1 and 2 and \$7 million for scenarios 3 and 4. For scenarios 1 and 3, the budget at year 3 is \$5 million, while for scenarios 2 and 4, the budget at year 3 is \$7 million.

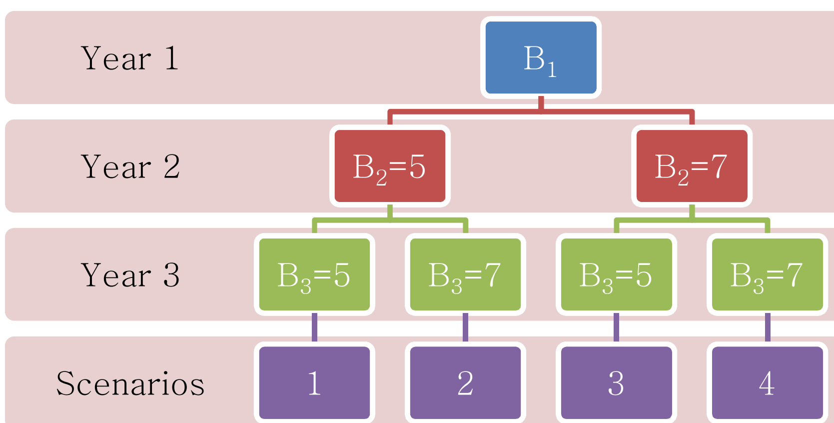

Figure 0-1 Scenario Tree

If scenarios n_1, n_2 ($n_1, n_2 \in \mathbf{N}$) have the same information state at time period t (sharing the same node at t in the scenario tree), the two scenarios are indistinguishable at t. In general, scenarios n_1, n_2 are indistinguishable at t if they are identical in realizations for all uncertain budgets up to time t. For example, in Figure 3-1, scenarios 1 and 2 are indistinguishable at year 2, as they have the same budget realization at that year. However, they are distinguishable at year 3, because their budgets at that year are different. Moreover, let $\mathbf{t}(n_1, n_2)$ denote the latest time period at the end of which scenarios n_1 and n_2 are indistinguishable. For example, in Figure 0-1, $\mathbf{t}(1,2) = 2$, scenarios 1 and 2 differ in terms of budget realization after year 2.

Using the notations discussed above, the infrastructure maintenance planning problem under budget uncertainty can be formulated as a multi-stage stochastic programming problem in Equations. (3.11)-(3.16).

$$\max \frac{1}{\sum_{s\in\mathbf{S}} L_s} \frac{1}{T+1} \left(\sum_{n=1}^{N} p^n \left(\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{m=1}^{M} L_s X_{s1mt}^n + \sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M} P_{si1m} L_s X_{simT}^n \right)\right) \tag{3.11}$$

$$\text{s.t.} \sum_{m=1}^{M} X_{sim1}^n = X_{si1}, \forall s\in \mathbf{S}, i\in \mathbf{I}, n\in \mathbf{N} \tag{3.12}$$

$$\sum_{m=1}^{M} X_{sjmt}^n = \sum_{m=1}^{M}\sum_{i=1}^{I} P_{sijm} X_{sim,t-1}^n, \forall s\in \mathbf{S}, j\in \mathbf{I}, t=2,...,T, n\in \mathbf{N} \tag{3.13}$$

$$\sum_{n=1}^{N}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M} C_{smt} X_{simt}^n L_s \le B_t^n, \forall s\in \mathbf{S}, j\in \mathbf{I}, t\in \mathbf{T}, n\in \mathbf{N} \tag{3.14}$$

$$0 \le X_{simt}^n \le 1, \forall s\in \mathbf{S}, i\in \mathbf{I}, m\in \mathbf{M}, t\in \mathbf{T}, n\in \mathbf{N} \tag{3.15}$$

$$X_{simt}^{n_1} = X_{simt}^{n_2}, \forall s\in \mathbf{S}, i\in \mathbf{I}, m\in \mathbf{M}, t\in \mathbf{t}(n_1,n_2), n_1\in \mathbf{N}, n_2\in \mathbf{N} \tag{3.16}$$

Objective (3.11) maximizes the expected annual proportion of facilities in the best condition state over the probability space of random variable $\tilde{B}_t$. Constraints (3.12)–(3.15) are the same as constraints (3.5)-(3.8) but for the nth scenario. Decisions for different scenarios are linked by nonanticipativity constraints (3.16). The nonanticipativity-constraint states that decision variables of scenario n_1 and n_2 are equal whenever n_1 and n_2 are indistinguishable. In stochastic programming, constraints enforcing such conditions are called nonanticipativity constraints, implying that the future cannot be anticipated. The nonanticipativity-constraint acts as a coupling constraint that connects different scenarios together and specifies how the information of budget is shared among scenarios. For example, in Figure 3-1, decision variables before year 3 should be the same for scenarios 1 and 2.

CHAPTER 4. SOLUTION PROCEDURE

Multi-stage stochastic programming is one of the most difficult problems in mathematical programming. The basic approach to multistage stochastic programs is to approximate the stochastic process using a process of finite scenarios exhibiting a tree structure. The size of the problem grows quickly as the number of stages and number of scenarios increase, typically leading to very large-scale linear programming models.

Existing computational methods for multistage stochastic programming problems include decomposition methods that exploit specific structures of the model to split it into manageable pieces and scenario reduction techniques that generate smaller scenario trees from an initial set of scenarios. Decomposition methods can be further classified into two groups: Primal decomposition methods that define subproblems according to time stages and dual methods that construct subproblems that correspond to scenarios.

In this research, the author proposes use of the augmented Lagrangian decomposition method (Rosa and Ruszczynski 1996) and scenario reduction method (Heitsch and Romisch 2009). The major computational advantage of the augmented Lagrangian decomposition method is the possibility of solving the dual problem by the multiplier method. Another important advantage of the augmented Lagrangian decomposition method over the usual Lagrangian duality is its sufficiency for primal recovery when the dual solution is known. The advantage of the scenario reduction method is that it significantly simplifies the computational effort. The following sections introduce the basic principle of these two methods.

4.1 Augmented Lagrangian Decomposition (ALD)

Let $X_1, X_2, ..., X_L$ be non-empty closed convex subsets, and let $f_i, i = 1,2,...,L,$ be convex functions. Moreover, let A_i be matrices of dimension $m \times n_i, i = 1,2,...,L,$ and let $b \in R^m$. Consider the following convex programming problem:

$$\min \sum_{i=1}^{L} f_i(x_i) \tag{4.1}$$

$$\sum_{i=1}^{L} A_i x_i = b \tag{4.2}$$

$$x_i \in X_i, i = 1,2,...,L \tag{4.3}$$

Problems (4.1)-(4.3) can be decomposed into L smaller and simpler problems $\left\{\min_{x_i \in X_i} f_i(x_i), i = 1,...,L\right\}$ if constraint (4.2) is relaxed. To use this special structure to solve the problem, the augmented Lagrangian function is defined for this problem as:

$$\Lambda(\mathbf{x}, \pi) = \sum_{i=1}^{L} f_i(x_i) + \pi\left(b - \sum_{i=1}^{L} A_i x_i\right) + \frac{\rho}{2}\left\| b - \sum_{i=1}^{L} A_i x_i \right\|^2 \tag{4.4}$$

where ρ is the penalty parameter and $\rho > 0$. The dual problem is also defined as:

$$\max_{\pi \in R^m} \left\{ g(\pi) = \inf_{x \in X} \Lambda(\mathbf{x}, \pi) \right\} \quad (4.5)$$

For every optimal solution $\hat{\pi}$ of (4.5), a point $\hat{\mathbf{x}}$ is a solution of (4.1)–(4.3) only if $\Lambda(\hat{\mathbf{x}}, \hat{\pi}) = \min_{\mathbf{x} \in \mathbf{X}} \Lambda(\mathbf{x}, \hat{\pi})$. Therefore, the optimal solution of problems (4.1)–(4.3) is obtained by solving the dual problem (4.5) instead (Ruszczynski 1997). The dual problem is solved by iteratively using the method of multipliers (4.6)–(4.7) until a convergence is reached (Sun and Yuan 2006):

$$\mathbf{x}^k = \arg\min_{\mathbf{x} \in \mathbf{X}} \Lambda(\mathbf{x}, \pi^k) \quad (4.6)$$

$$\pi^{k+1} = \pi^k + \rho(b - \mathbf{A}\mathbf{x}^k), k = 0, 1, 2, \cdots \quad (4.7)$$

where k is the iteration counter for the method of multipliers.

Thus far, although the coupling constraint (4.2) is relaxed, solving (4.6) is still cumbersome, because the third term of (4.4) is inseparable. As a result, problem (4.6) cannot be split into smaller subproblems for $x_i, i = 1, 2, ..., L.$ To overcome this difficulty, an iterative nonlinear Jacobi method to the minimization of (4.4) is applied (Ruszczynski 1997; Rosa and Ruszczynski 1996). This method uses a certain approximation of the minimizer $\mathbf{x}^k$ in (4.6) and solves the following simplified functions for $i = 1, 2, ..., L$:

$$\Lambda_i(x_i, \tilde{\mathbf{x}}_i, \pi) = f_i(x_i) - A_i^T \pi x_i + \frac{\rho}{2} \left\| b - A_i x_i - \sum_{j \neq i} A_j x_j \right\|^2 \quad (4.8)$$

where $\tilde{\mathbf{x}}_i$ represents all the solutions x_j with $j \neq i$. The main goal of this approach is to replace (4.6) with L smaller problems:

$$\min_{x_i \in X_i} \Lambda_i(x_i, \tilde{\mathbf{x}}_i, \pi^k), i = 1, 2, ..., L \quad (4.9)$$

and to iteratively update the parameter $\tilde{\mathbf{x}}_i, i = 1, 2, ..., L$. In this sense, solving (4.9) is equivalent to solving (4.6) with respect to x_i while keeping all $x_j, j \neq i$ fixed. In this way, (4.6) can be solved using the Jacobi method (Rosa and Ruszczynski 1996):

Step 0: Set the iteration counter of the Jacobi method $r := 0$ and determine the initial solution values $\mathbf{x}^{k,0} = \mathbf{x}^{k-1}$.

Step 1: Set $r := r + 1$. Solve (5.9) for $i = 1, 2, ..., L$ and obtain the solution $\mathbf{x}^{k,r}$, where $\mathbf{x}^{k,r} = \{x_1^{k,r}, x_2^{k,r}, ..., x_L^{k,r}\}$.

Step 2: If $\mathbf{A}\mathbf{x}^{k,r} = \mathbf{A}\mathbf{x}^{k,r-1}$ then stop and set $\mathbf{x}^k = \mathbf{x}^{k,r}$; otherwise update $\mathbf{x}^{k,r}$ by (5.10) and go to Step 1:

$$\mathbf{x}^{k,r} = \mathbf{x}^{k,r-1} + \tau(\mathbf{x}^{k,r} - \mathbf{x}^{k,r-1}) \quad (4.10)$$

where τ is a weighting factor.

4.2 Application of ALD to Stochastic Programming

Using the idea discussed in the last section, infrastructure maintenance planning problems under budget uncertainty (3.11)-(3.16) can be decomposed into N subproblems (N scenarios), if the nonanticipativity constraint (3.16) is relaxed. Using the Figure 4-1 as an example, if the nonanticipativity constraint is relaxed, the scenario tree will be separated to individual branches (Figure 0-1).

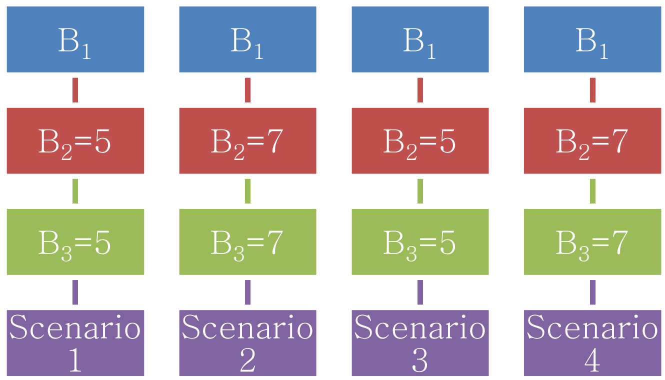

Figure 0-1 Scenario Tree after Decomposition

Because of the special structure of the problem, the augmented Lagrangian decomposition method can be used. The augmented Lagrangian function Λ is first defined as:

$$\Lambda(x,\pi)=\frac{1}{\sum_{s\in \mathbf{S}}L_s}\frac{1}{T+1}\left(\sum_{n=1}^{N}p^n\left(\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{m=1}^{M}L_sX^n_{s1mt}+\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}P_{i1m}L_sX^n_{simT}\right)\right)$$
$$-\sum_{n=1}^{N}\sum_{n'=1}^{N}\sum_{t=1}^{\mathbf{t}(n,n')}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}\pi^{n,n'}_{stim}\left(X^n_{stim}-X^{n'}_{stim}\right)+\frac{\rho}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N}\sum_{t=1}^{\mathbf{t}(n,n')}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}\left(X^n_{stim}-X^{n'}_{stim}\right)^2 \quad (4.11)$$

The subproblem of the nth scenario is expressed as:

$$\max \frac{1}{\sum_{s\in \mathbf{S}}L_s}\frac{1}{T+1}p_s\left(\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{m=1}^{M}L_sX^n_{s1mt}+\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}P_{i1m}L_sX^n_{simT}\right)$$
$$-\sum_{n=1}^{N}\sum_{n'=1}^{N}\sum_{t=1}^{\mathbf{t}(n,n')}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}\pi^{n,n'}_{stim}X^n_{stim}+\frac{2}{\rho}\sum_{n=1}^{N}\sum_{n'=1}^{N}\sum_{t=1}^{\mathbf{t}(n,n')}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}\left(X^n_{stim}\right)^2 \quad (4.12)$$
$$-\frac{\rho}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N}\sum_{t=1}^{\mathbf{t}(n,n')}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M}2X^n_{stim}\tilde{X}^{n'}_{stim}$$

$$\text{s.t.} \quad \sum_{m=1}^{M}X^n_{sim1}=X_{si1}, \forall s\in \mathbf{S}, i\in \mathbf{I}, \quad (4.13)$$

$$\sum_{m=1}^{M} X_{sjmt}^{n} = \sum_{m=1}^{M}\sum_{i=1}^{I} P_{sijm} X_{sim,t-1}^{n}, \forall s \in \mathbf{S}, j \in \mathbf{I}, t = 2,...,T \quad (4.14)$$

$$\sum_{n=1}^{N}\sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{m=1}^{M} C_{smt} X_{simt}^{n} L_s \le B_t^n, \forall s \in \mathbf{S}, i \in \mathbf{I}, t \in \mathbf{T} \quad (4.15)$$

$$0 \le X_{simt}^{n} \le 1, \forall s \in \mathbf{S}, i \in \mathbf{I}, m \in \mathbf{M}, t \in \mathbf{T} \quad (4.16)$$

The problem (4.12)-(4.16) is minimized with respect to decision variables associated with the n th scenario assuming that decision variables of other scenarios are temporarily fixed. As suggested by Rosa and Ruszczynski (1996), scenarios are numbered so that at the ith scenario, the $i+1$ th scenario has the largest last common stage with i among all scenarios $j > i$. The augmented Lagrangian decomposition algorithm is carried out in the order of 1 to N. By applying the method of multipliers and the Jacobi method, the infrastructure maintenance planning problem is solved.

4.3 Scenario Reduction (SR)

Scenario reduction consists in eliminating scenarios that are similar to other scenarios. In the beginning, a large number of scenarios exist. These scenarios normally result from a simulation where the distribution of the simulated random variable is known. The aim of the scenario reduction is that a reduced number of scenarios still represent the underlying distribution in an acceptable way.

Assume that the original probability distribution P is discrete and carried by finitely many scenarios $\omega_i \in \Omega$ with weights $p_i > 0, i = 1,\ldots,N$, and $\sum_{i=1}^{N} p_i = 1$, i.e., $P = \sum_{i=1}^{N} p_i \delta_{\omega_i}$. Let $J \subset \{1,\ldots,N\}$ and consider the probability measure Q having scenario ω_j with probabilities q_j, $j \in \{1,\ldots,N\} \setminus J$, i.e., compared to P the measure $Q = \sum_{j \notin J} q_j \delta_{\omega_j}$ is reduced by deleting all scenarios $\omega_j, j \in J$ and by assigning new probabilistic weights q_j to each scenario $\omega_j, j \notin J$. One of the algorithms of reducing scenarios is to delete one scenario at a time. Therefore, the optimal deletion problem is

$$\min_{l \in \{1,\ldots,N\}} p_l \min_{j \ne l} c\left(\omega_l, \omega_j\right) \quad (4.17)$$

If the minimum is attained at $l_* \in \{1,\ldots,N\}$, i.e., the scenario ω_{l_*} is deleted, the optimal redistribution rule is $\overline{q}_l = p_l$ for each $l \notin \{l_*, j(l_*)\}$ and $\overline{q}_{j(l_*)} = p_{j(l_*)} + p_{l_*}$, where $j(l_*) \in \arg\min_{j \ne l_*} c\left(\omega_{l_*}, \omega_j\right)$. The optimal deletion of a single scenario will be repeated recursively until a prescribed number k of scenarios is deleted.

CHAPTER 5. CASE STUDY

A numerical experiment applying the proposed methodology to an example problem of a road network is carried out in the case study. The characteristics of the test problem and some implementation details are introduced. The benefit of using the stochastic programming approach over a deterministic approach is discussed. The computational result is commented and the proposed algorithm is examined in terms of trade-offs between computational effectiveness and solution quality. Test runs were programmed in MATLAB and performed on a standard desktop computer with 1 GB of memory and a 3.4 GHz CPU.

5.1 Case Study Data Set

The road network in Dallas District is used for the case study with data taken from the Texas Department of Transportation (TxDOT) Pavement Management Information System (PMIS). The PMIS is an automated system for storing, retrieving, analyzing, and reporting pavement condition information. It can be used to retrieve and analyze pavement information to compare maintenance and rehabilitation treatment alternatives, monitor current pavement conditions, and estimate total pavement needs. The main characteristics of the Dallas District road network are presented as follows.

5.1.1 Size of the Network

In the PMIS database for the Dallas District, there are five different functional class highways: Business Road (BR), Farm to the Market (FM), Interstate Highway (IH), State Highway (SH) and US Highway (US). According to their similarities in terms of the deterioration pattern, the highways are grouped into three broader categories as presented in Table 5.1.

Table 5.1 Road Network Length

Highway Groups	Length (Lane-Kilometers)
Group I (IH, US and BR)	8299
Group II (SH)	3104
Group III (FM)	5045

5.1.2 Planning Horizon

The objective of the case study is to develop a five-year maintenance plan for the road network, where the maintenance treatments will be applied at the beginning of each year.

5.1.3 Performance Indicator

In this case study, the Condition Score (CS) in the PMIS database is used as the performance indicator. The TxDOT PMIS stores three scores that represent the general condition of a pavement (TxDOT 2000), The Distress Score (DS) reflects the amount of visible surface deterioration of a pavement, with a range from 1 (the most distress) to 100 (the least distress). The Ride Score (RS) is a measure of the pavement's roughness, ranging from 0.1 (the roughest) to 5.0 (the smoothest). The Condition Score represents the pavement's overall condition in terms of both distress and ride quality ranging from 1 (the worst condition) to 100 (the best condition). The condition of a pavement is discretized into five different states according to its condition score (Table 5.2)

Table 5.2 PMIS Condition Scores

Condition Score	**Description**
90-100	Very Good
70-89	Good
50-69	Fair
35-49	Poor
1-34	Very Poor

The initial condition of the road network in terms of the percentage in each condition state is shown in Table 5.3. The numbers in this table represent the percentage of the corresponding road type in a specific condition state. For example, 73 percent of Type I road pavements—which comprise the majority of the road network—are in "Very Good" condition.

Table 5.3 Road Network Initial Condition (%)

Condition State\Road Groups	**IH, US and BR**	**SH**	**FM**
Very Good	73	58	62
Good	11	15	16
Fair	7	10	10
Poor	5	9	8
Very Poor	4	8	4

The goal of the road network's five year maintenance plan is that 90 percent of the road group I should be in "Very Good" condition state, and 80 percent of road groups II and III should be in "Very Good" condition state as shown in Table 5.4.

Table 5.4 Road Condition Requirements

Condition State\Road Groups	**I**	**II**	**III**
Very Good (100-80)	90%	80%	80%

5.1.4 Transition Probability

Generally, there are two ways that transition probability can be estimated. The first way is by simulation through pavement design equations (Gao and Zhang 2007), while the second way estimates the probability through historical data (Butt et al. 1987; Jiang et al. 1989; Wang et al. 1994). In this case study, the transition probability for each of the road groups is calculated based on the historical data from the Dallas PMIS database. The results are shown in Table 5.5, Table 5.6, Table 5.7.

Table 5.5 TPM for Road Group I

Initial State\Next State	Very Good	Good	Fair	Poor	Very Poor
Very Good	0.85	0.10	0.03	0.01	0.00
Good	0.00	0.57	0.28	0.12	0.04
Fair	0.00	0.00	0.47	0.39	0.13
Poor	0.00	0.00	0.00	0.56	0.44
Very Poor	0.00	0.00	0.00	0.00	1.00

Table 5.6 TPM for Road Group II

Initial State\Next State	Very Good	Good	Fair	Poor	Very Poor
Very Good	0.74	0.16	0.07	0.03	0.01
Good	0.00	0.35	0.37	0.21	0.07
Fair	0.00	0.00	0.45	0.44	0.11
Poor	0.00	0.00	0.00	0.55	0.45
Very Poor	0.00	0.00	0.00	0.00	1.00

Table 5.7 TPM for Road Group III

Initial State\Next State	Very Good	Good	Fair	Poor	Very Poor
Very Good	0.77	0.14	0.06	0.03	0.01
Good	0.00	0.36	0.39	0.19	0.06
Fair	0.00	0.00	0.38	0.43	0.19
Poor	0.00	0.00	0.00	0.41	0.59
Very Poor	0.00	0.00	0.00	0.00	1.00

5.1.5 Maintenance Effect

Maintenance treatments could be at any level, from the least expensive in preventive maintenance to the most expensive in reconstruction. However, it is not necessary for programming at the network level to be as detailed as at the project level. Four maintenance treatments levels are used in this case study: Do Nothing, Preventive Maintenance, Light Rehabilitation and Heavy Rehabilitation. For a given section at any given year, four possible treatments can be performed. Preventive maintenance, including seal coat, micro-surfacing or thin overlay, is aimed at extending the life of bituminous surfaces by retarding the effects of weathering and aging before significant amounts of distress have occurred. Rehabilitation involves heavier treatmetns intended to increase the structural capacity, restore ride and seal the base and subgrade layers. For demonstration purpose, the assumed maintenance treatments effect for a pavement section is given in Table 5.8.

Table 5.8 Maintenance Treatments Effect

M&R treatment	Condition state before treatments	Condition state after treatment
Do Nothing	Very Good	Very Good
	Good	Good
	Fair	Fair
	Poor	Poor
	Very Poor	Very Poor
Preventive Maintenance	Very Good	Very Good
	Good	Very Good
	Fair	Fair
	Poor	Poor
	Very Poor	Very Poor
Light Rehabilitation	Very Good	Very Good
	Good	Very Good
	Fair	Very Good
	Poor	Good
	Very Poor	Fair
Heavy Rehabilitation	Very Good	Very Good
	Good	Very Good
	Fair	Very Good
	Poor	Very Good
	Very Poor	Very Good

5.1.6 Maintenance Cost (Agency Cost)

The unit costs for all types of treatments are taken from the work of Wang et al. (2003) as listed in Table 5.9.

Table 5.9 Maintenance Treatment Costs

Road Group	Maintenance Treatment	Maintenance treatment unit cost ($1000/lane/km)
I	Do Nothing	0
	Preventive Maintenance	10
	Light Rehabilitation	100
	Heavy Rehabilitation	500
II	Do Nothing	0
	Preventive Maintenance	8
	Light Rehabilitation	80
	Heavy Rehabilitation	400
III	Do Nothing	0
	Preventive Maintenance	5
	Light Rehabilitation	20
	Heavy Rehabilitation	100

5.2 Solution of ALD

In this research, it is assumed that the budget at every year is unknown but will be allocated from $80 million, $100 million, and $120 million with equal probabilities. Therefore, there are a total of 243 scenarios for this problem. The characteristics of the original problem and the subproblems after decomposition are summarized in Table 5.10. Before decomposition, the stochastic programming problem has 287,955 constraints and 24,300 variables, which makes it impossible to be solved on a standard desktop computer. The subproblems require much less computational effort with each subproblem having only 75 constraints and 300 variables. As noted, by adopting the decomposition technique, the size of the problem is dramatically reduced.

Table 5.10 Computational Characteristics

	Subproblem after applying augmented Lagrangian decomposition	Original problem
Number of constraints	75	287,955
Number of variables	300	24,300

A stopping criterion $\varepsilon = 10^{-3}$ is used for both the method of multipliers and the Jacobi method. The value of ρ and τ is set at 0.5. The initial values of the decision variable $\mathbf{x}^0$ are assigned zeros. Figure 5-1 shows the relationship between the iteration of the method of multipliers and the iteration of the Jacobi method. As illustrated, the Jacobi method occurs with greatest frequency at the beginning of the algorithm, then the iteration of Jacobi steps decreases rapidly.

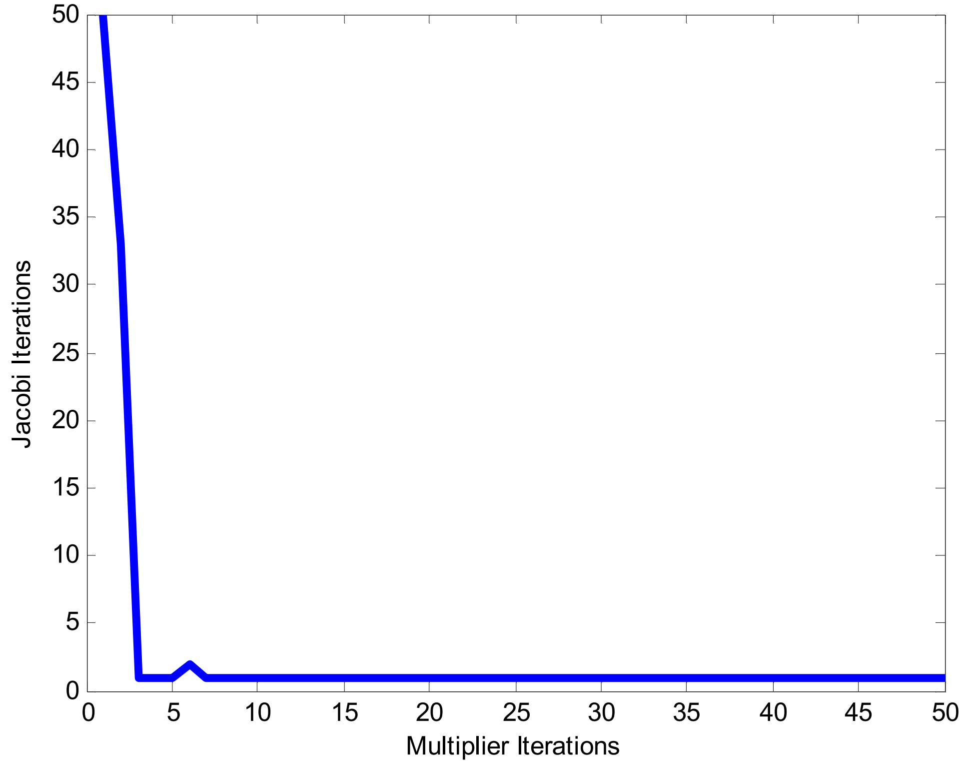

Figure 5-1 Number of Jacobi Steps in Each Outer Loop

Figure 5-2 shows the relationship between the multiplier iterations and the number of nonanticipativity constraints violated. As seen in this figure, the constraints violation drops quickly during the first four multiplier iterations; then it is subsequently followed by a slower convergence until the stopping criterion is reached. The optimal objective function value of the stochastic programming approach is obtained as 78.23.

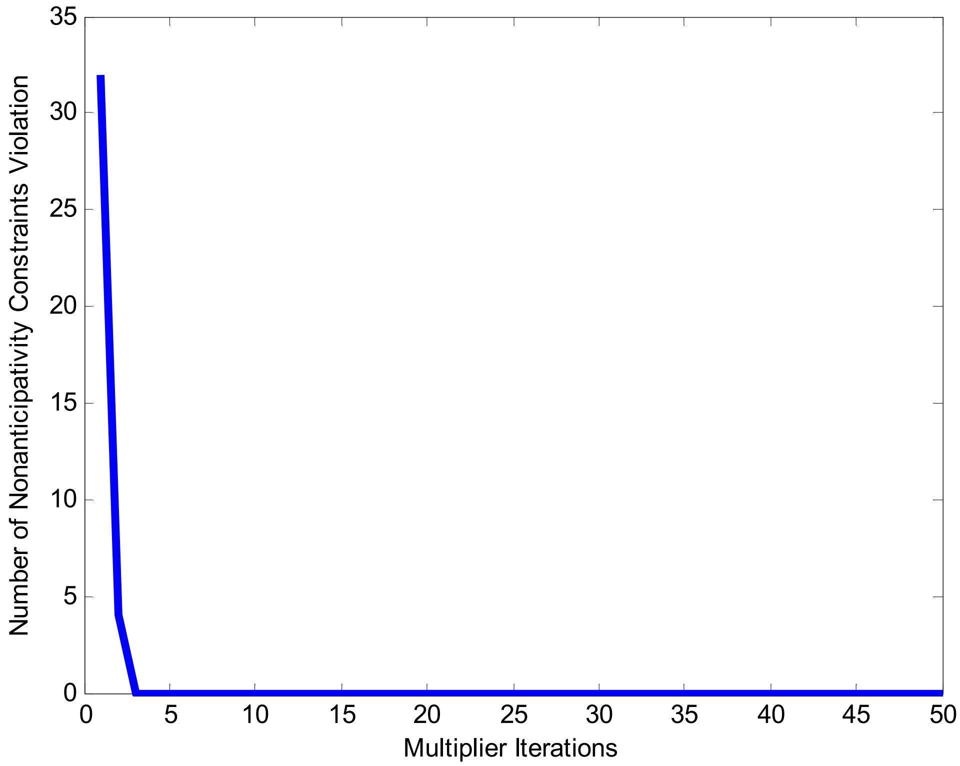

Figure 5-2 Number of Violated Nonanticipativity Constraints

5.3 Deterministic Solution (EV)

An alternative to the stochastic programming (SP) approach is to consider only the expected budget values, which is known as the expected value (EV) approach. This approach is to schedule the maintenance activities assuming that the budget will take their expected values during the planning horizon. The EV approach can be mathematically expressed as

$$EV = \min_x z\left(x, \bar{\xi}\right) \tag{5.1}$$

where x represents the decision variables, z represents the objective function and $\bar{\xi}$ is the expected value of the random variable ξ. Using the example of Figure 0-1, the concept of the EV approach can be illustrated in Figure 5-3.

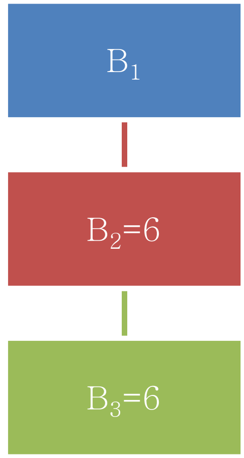

Figure 5-3 Scenario Tree of EV Approach

The advantage of this approach is that it is computationally easy to solve. In this research, by solving the deterministic problem (3.4)-(3.8), the detailed maintenance plan is obtained as shown in Table 5.11. The numbers in the table are the percentage of whole road network that will receive the corresponding maintenance treatments.

Table 5.11 Maintenance Plan of Deterministic Solution

Year	Do Nothing	Preventive Maintenance	Light Rehabilitation	Heavy Rehabilitation
1	0.257	0.670	0.073	0.000
2	0.107	0.869	0.017	0.008
3	0.100	0.893	0.000	0.007
4	0.084	0.880	0.029	0.007
5	0.142	0.815	0.038	0.005

The objective function of the EV approach is obtained as 89.97, which is much better than the SP solution 78.23. This is no surprise, since the EV approach only considers one scenario while the SP considers all 243 scenarios. The EV result actually represents the upper bound of the SP problem. However, ignoring the random characteristics of future budget may lead to suboptimal result. The EV solution is infeasible (in terms of budget constraint satisfaction) to some of scenarios. As a result, some of the planned maintenance activities may have to be canceled and a new maintenance plan has to be made. In order to evaluate the benefit of using the SP method against the EV approach quantitatively, the EV solution $\bar{x}\left(\bar{\xi}\right)$ is used to calculate the expected objective function value for all possible scenarios. The resulted quantity is called expected result of using the EV solution (EEV).

$$EEV = E_{\xi}\left(z\left(\bar{x}\left(\bar{\xi}\right),\xi\right)\right) \qquad (5.2)$$

EEV measures how $\bar{x}(\bar{\xi})$ performs, allowing subsequent-stages decisions to be chosen optimally. In other words, EEV represents the expected objective function value if decisions are made ignoring the budget uncertainty. By using (5.2), the EEV of the test problem can be calculated as 67.35. The difference between the EEV and the SP solution is called value of the stochastic solution (VSS),

$$VSS = EEV - SP \tag{5.3}$$

A small VSS means that the approximation of the SP by the EV approach is applicable. For the test example, however, VSS is almost 15% of the value of SP, which confirms that there is an obvious benefit in using a stochastic model than a deterministic one.

In order to identify the difference between SP and EV, Table 5.12 compares the maintenance plans of them at the first year. As can be seen in Table 5.12, more resources are allocated to preventive maintenance in the stochastic programming approach. Therefore, the stochastic solution alleviates the effect of possible funding shortages by allocating more resources to preventive maintenance treatments. The underlying strategy of the stochastic solution is to spread out current funding among more pavement sections given the existence of budget uncertainty in future years. Using this strategy, the expected condition of a road network can be optimized. In practice, the proposed stochastic programming problem must be solved every year when decision makers become aware of specific appropriations and budget constraints. The maintenance plan obtained for the first year can be used to schedule activities during the year under consideration.

Table 5.12 Maintenance Plan Comparison of Year 1 between EV and SP

Method	Do Nothing	Preventive Maintenance	Light Rehabilitation	Heavy Rehabilitation
EV	0.257	0.670	0.073	0.000
SP	0.170	0.780	0.050	0.000

5.4 COMPUTATIONAL COMPARISON (ALD, EV AND SR)

To demonstrate the effectiveness of the proposed decomposition algorithm, a computational comparison is carried out. Table 5.13 lists the computational characteristics of the augmented Lagrangian decomposition (ALD) method, scenario reduction (SR) method and the EV approach for problems (3.11)–(3.16). The scenario reduction method is another approach to solve the stochastic programming problem. It generates a scenario subset closest to the initial distribution in terms of a natural probability metric. In other words, only a portion of the original scenarios are selected in SR to reduce the size of the problem. As can be seen in Figure 5-4 and Figure 5-5, the size of the problem can be largely reduced after applying the scenario reduction method.

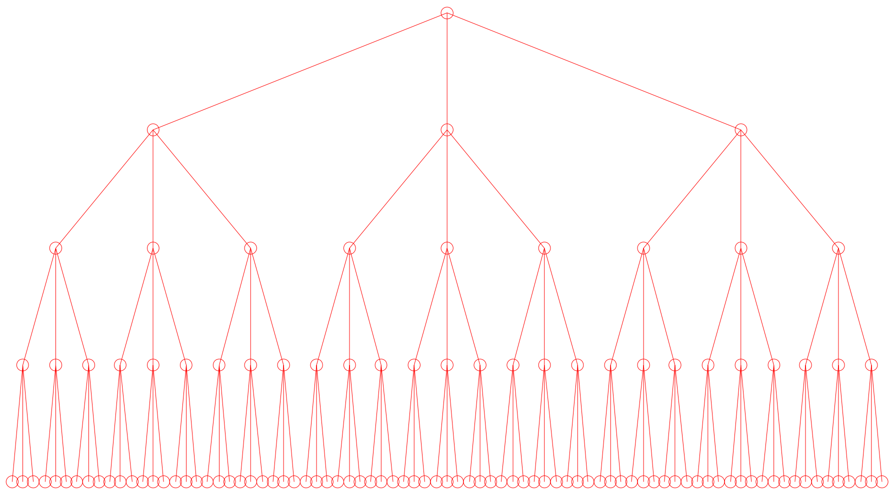

Figure 5-4 Scenario Tree before Reduction

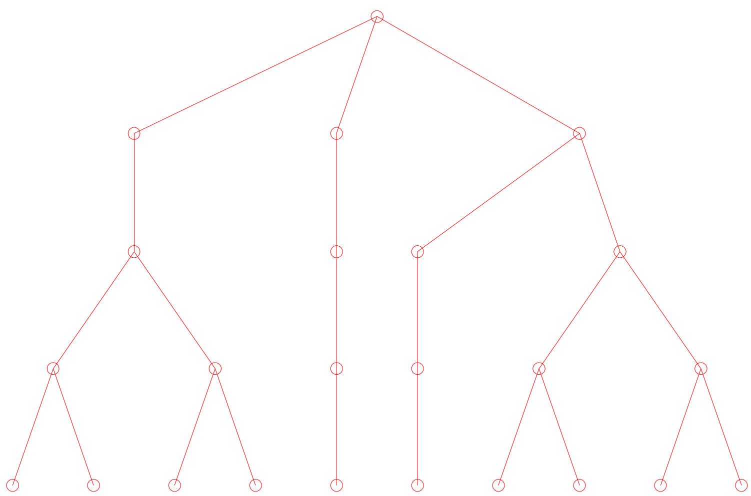

Figure 5-5 Scenario Tree after Reduction

As shown in Table 5.13, the EV approach and the SR approach are much faster than the ALD method in terms of computational time. Because of the reduction of uncertainty, the objective function values of SR and EV are higher than the result of ALD. However, as shown in the fourth column of Table 5.13, by using the idea of (5.2), the ALD approach produces the best expected objective function value for all 243 scenarios. This is because the ALD approach takes all scenarios into consideration at the beginning of the planning horizon; and the solution of ALD consists of maintenance plan for every scenario. However, the solutions obtained from SR and EV considers only part of the scenarios. As a result, some of the planned maintenance activities may have to be re-planed in the future, which makes the solution suboptimal. It is up to the decision maker's choice to make trade-offs of solution quality and computational effort.

Table 5.13 Computational Characteristics Comparison

Methods	Computational Time (seconds)	Objective Function Value	Expected Objective Function value
ALD	240.1398	78.23	78.23
SR(reduced to 10 scenarios)	7.6875	80.66	72.49
EV	0.5938	89.97	67.35

CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

6.1 SUMMARY

The main objective of this study is to define a methodological framework for infrastructure asset management maintenance scheduling problem under budget uncertainty and to develop solution algorithms for solving the defined problem. A multistage linear stochastic programming model is developed and the effectiveness and efficiency of three different solution approaches are investigated. Finally, the developed model and solution algorithms are used to solve practical case.

6.2 CONCLUSIONS

Conclusions drawn from this study are as follows:

1. Stochastic programming methods can be used to model the uncertainty of future maintenance budgets as random variables in infrastructure asset management resource allocation optimization problems. Stochastic programming is based on probability theory and mathematical programming. A Stochastic Programming problem, that is defined by chance constraint functions and a probabilistic objective function, can be translated to a deterministic optimization problem by defining a scenario tree. However, as the number of planning stages and number of scenarios at each stage increase, the size of the resulting deterministic problem increases quickly. Three different approaches (ALD, EV and SR) are investigated in this research. The ALD approach is able to produce the best results.
2. A road network case is studied as part of this research. The following findings indicate that the proposed model and solution procedure is able to solve the maintenance scheduling problem efficiently and effectively. The benefit of using the stochastic programming approach over a deterministic approach is also discussed. Stochastic programming solutions, which take future budget uncertainty into consideration, tend to allocate more resource into preventive maintenance than deterministic solution that ignores the uncertainty information. The proposed methodology can help decision makers effectively obtain optimal maintenance planning under budget uncertainty.

6.3 RECOMMENDATIONS FOR FUTURE RESEARCH

In the following, some areas are given with respect to opportunities for future research.

6.3.1 Stochastic Integer Programming

The current framework is based on stochastic linear programming, where the decision variables determine the percentage of infrastructure system receiving a certain type of maintenance treatment. As discussed in section 2.2.2, this formulation has its advantage that the solution is guaranteed to be global optimal. However, this approach simplifies the decision making process by giving maintenance plans for "groups" instead of individual facilities. Therefore, an agency which manages an infrastructure system has to further allocate resource from "groups" to specific facilities after running the linear programming model. In other words, the linear programming approach may not give the best plan compared with Integer Programming formulation, where the decision variables directly specify the location, timing and treatment type. As discussed before, the disadvantage of the IP approach is that the size of the problem increase exponentially as the number of facilities, the number of planning stages and the number of maintenance treatments increase. It is interesting to search solution algorithms that will solve large-scale IP formulation for infrastructure maintenance scheduling problem and its application in the stochastic setting.

6.3.2 Uncertainties other than Budget

There are other uncertainties in the infrastructure management process. For example, infrastructure deterioration is a dynamic, complicated, and stochastic process affected by a variety of factors such as usage, environmental conditions, and structural capacities, as well as certain unobserved factors. Hence, the performance of an infrastructure facility can never be predicted with absolute certainty. Ignoring such uncertainties during the modeling process may compromise the validity of an optimal solution. It is also important to take those uncertainties into consideration when making maintenance resource allocation decisions.

6.3.3 Different Ownership

In the current framework, the developed model is suitable for government agencies like State DOTs. In recent years, public private partnership (PPP) is becoming an increasingly popular method of funding large infrastructure projects. These PPP projects involve financing for different stages of a project including the design, build, expansion, upgrade and operation. This use as a relatively new source of funding infrastructure projects has highlighted some of the challenges and issues when planning maintenance activities. Therefore, it is important that this new change being reflected in the maintenance planning model.

6.3.4 Balance between different regions

In this report, the developed model can help decision makers allocate funds to infrastructure facilities under management. However, for some agencies, another factor is important that the balance between different regions or districts have to be taken into consideration. For example, in Texas Department of Transportation, funds have to be distributed to 25 districts and the districts can further allocate it to specific projects. Therefore, it is important to incorporate this factor as additional constraints to the model, allowing it to make maintenance plans by considering different condition and demands among districts.

6.3.5 Multiple objectives

In the current methodology framework, only one objective is considered in the optimization problem formulation. Single-objective optimization is adequate if the decision maker is satisfied with optimizing only one objective. In practice, there may be more than one objective that need to be optimized in the infrastructure maintenance planning process. Different competing objectives may have significantly different impacts on the resulting solutions. For example, an agency may wish to find maintenance strategies that maximize system conditions while also minimizing the risk of safety failures. A trade-off compromise can be used to either optimize one objective and include the competing objectives as constraints, or optimize the sum of the competing objectives. In future research works, these multi-objective issues should be addressed.

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