Paper No. 03-4340

# Dynamic Origin–Destination Demand Estimation with Multiday Link Traffic Counts for Planning Applications

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**A dynamic origin–destination demand estimation model for planning applications with real-time link counts from multiple days is presented. Based on an iterative bilevel estimation framework, the upper-level problem is to minimize both the deviation between estimated link flows and real-time link counts and the deviation between estimated time-dependent demand and given historical static demand. These two types of deviations are combined into a weighted objective function, where the weighting value is determined by an interactive approach to obtain the best compromise solution. The single-day formulation is further extended to use link counts from multiple days to estimate the variation in traffic demand over multiple days. A case study based on the Irvine test bed network is conducted to illustrate the methodology and estimate day-to-day demand patterns. The application illustrates considerable benefits in analyzing the demand dynamics with multiday data.**

The deployment of intelligent transportation systems (ITS) offers considerable opportunity to acquire vast amounts of real-time traffic data that can contribute to improved understanding of traffic dynamic processes as well as a richer basis for the management of transportation systems. Specifically, real-time traffic link counts (or occupancies obtained from loop detectors or other sensors) can be an important data source to estimate time-dependent origin–destination (O-D) travel demand. In the last two decades, significant progress has been made on the dynamic O-D estimation problem with real-time link counts. Early research (*1*) proposed methods to estimate time-dependent demand on individual components such as a single intersection or a freeway corridor. Estimators for dynamic O-D demand in a general network using a simplified assignment model were proposed by Cascetta et al. (*2*). Growing interest in the application of simulation-based dynamic traffic assignment (DTA) models has been accompanied by research into the estimation of dynamic O-D matrices. A recent approach uses a bilevel generalized least-squares optimization framework (*3, 4*) for this problem, while seeking to maintain internal consistency of the demand-dependent link-flow proportions between the upper-level demand estimation problem and the lower-level DTA problem. Extensive literature review of the dynamic O-D demand estimation problem and its inherent connection to DTA can be found elsewhere (*4, 5*).

Most existing procedures for dynamic O-D estimation focus on real-time applications, where computational considerations often limit the choice of methodology (*6–8*). In contrast, the dynamic O-D demand estimation problem in the transportation planning context has not received adequate attention, even though the time-dependent O-D demand matrix is a key input for successful application of DTA in planning practice. The requirements of planning applications for dynamic O-D estimation are different from those of real-time operational applications.

First, the primary concern in planning applications is to improve the final quality of O-D demand tables through effective utilization of multiple sources of information. These include any available historical static demand matrix, coupled with real-time traffic data (e.g., link counts, observed travel time) as well as the planners' knowledge. Furthermore, the O-D demand estimation problem in planning applications usually deals with a large-scale urban network that may have thousands of links and hundreds of traffic analysis zones. Most importantly, real-time traffic information may be available only on a subset of links in the study network, which increases the difficulty in inferring the dynamic O-D demand table for all the O-D pairs. Effective utilization of real-time data together with other information sources to estimate reliable time-dependent O-D demand is an important and challenging problem in current planning practice.

On the other hand, once installed, loop detectors can record multiple days of link counts continuously at minimal additional cost, thereby providing the opportunity to estimate the day-to-day demand variations in a cost-effective way. The day-to-day variability in demand is an important consideration for certain demand management policies (e.g., high-occupancy vehicle pricing) and traffic management (e.g., signal control), and an O-D demand matrix considering day-to-day uncertainty is an essential input for stochastic dynamic traffic assignment (*9*). Previous research has relied on survey data (*10*), which might suffer from low response rates and attrition due to the extended survey horizon (necessary to obtain multiday responses). Another possible information source is use of Global Positioning System (GPS) devices (possibly embedded in a mobile phone) to track and record the complete travel data for sampled individuals (*11*). However, current equipment cost for network level deployment would likely be too high.

In this paper, a dynamic O-D estimation model that explicitly considers the historical static O-D demand matrix is proposed, and then several possible strategies for ensuring that the problem is identifiable are discussed. The next section focuses on the estimation of day-to-day demand variations by using multiday link counts as well as the related hypothesis testing. Finally, a case study is presented with real-time link counts on the Irvine network.

## MODEL WITH ONE-DAY OBSERVATIONS

### Model Framework

The model presented here is an extension of the iterative bilevel estimation framework proposed by Tavana and Mahmassani (*3*). Specif-

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ically, the upper-level problem is a constrained ordinary least-squares problem, which is to estimate the dynamic O-D demand based on given link-flow proportions. The link-flow proportions in turn are generated from the dynamic traffic network loading problem at the lower level, which is solved by a DTA simulation program—namely, DYNASMART-P (*12*).

The following notation is used to represent all the variables in the demand estimation formulation. In this section, the concern is only with demand estimation using one-day link counts, so the subscript of day $m$ is dropped for simplicity.

- $h$ = subscript for the observation intervals, during which the traffic volume is accumulated and reported, $h = 1, \ldots, H$.
- $H$ = number of observation time intervals in estimation period.
- $l$ = subscript for links with traffic flow measurements, $l = 1, \ldots, L$.
- $L$ = number of links in the network that have flow measurements.
- $t$ = subscript for aggregated departure time intervals, $t = 1, \ldots, T$.
- $T$ = number of aggregated departure time intervals in estimation period.
- $i$ = subscript for origin zone, $i = 1, \ldots, I$.
- $I$ = number of origin zones in network.
- $j$ = subscript for destination zone, $j = 1, \ldots, J$.
- $J$ = number of destination zones in network.
- $m$ = subscript for day of week.
- $M$ = number of days of week, $m = 1, \ldots, M$. In this paper, $MM = 5$, representing Monday through Friday.
- $c_{(l,h),m}$ = measured traffic volume on link $l$, during observation interval $h$, on day $m$.
- $C_m$ = vector of measured flows on the links, consisting of element $c_{(l,h),m}$.
- $d_{(t,i,j),m}$ = demand volume with destination in zone $j$, originating their trip at zone $i$ during aggregated departure interval $t$ on day $m$.
- $D_m$ = vector of O-D demand flows, consisting of elements $d_{(t,i,j),m}$ on day $m$.
- $p_{(l,h),(t,i,j),m}$ = link-flow proportions—that is, proportion of demand flow $d_{(t,i,j),m}$ that flows onto link $l$ during observation interval $h$.
- $P_m$ = matrix of link-flow proportions, consisting of element $p_{(l,h),(t,i,j),m}$.
- $\epsilon_{(l,h),m}$ = combined error terms in estimation of traffic flow on link $l$ during observation interval $h$ on day $m$.
- $E_m$ = vector of combined error terms, consisting of elements $\epsilon_{(l,h),m}$ for link flow.
- $g_{(i,j)}$ = target demand, which is the total traffic demand during period of interest for each O-D pair $(i, j)$.
- $G$ = target demand vector, which is a vector of total traffic demand during period of interest, consisting of elements $g_{(i,j)}$.
- $\eta_{(i,j),m}$ = combined error terms in estimation of total traffic demand during period of interest from zone $i$ to zone $j$ on day $m$.
- $A$ = mapping matrix between time-dependent demand and total demand.
- $\Pi_m$ = vector of combined error terms, consisting of elements $\eta_{(i,j),m}$ for total traffic demand during period of interest.

Two objectives are considered in this formulation. The first one is to minimize the deviation between observed link flows and estimated link flows, as indicated in Equations 1*a* and 1*b*. The second objective is to minimize the deviation between the target demand and estimated demand. Suppose the target demand is a historical static demand table for the entire study horizon, so the second objective function can be explicitly written as the difference between the static demand and the sum of dynamic demand over the study period, as indicated in Equations 2*a* and 2*b*.

$$C = P \times D + E \tag{1a}$$

or

$$c_{(l,h)} = \sum_{t,i,j} p_{(l,h),(t,i,j)} d_{(t,i,j)} + \epsilon_{(l,h)} \tag{1b}$$

$$G = A \times D + \Pi \tag{2a}$$

or

$$g(i, j) = \sum_{t} d_{(t,i,j)} + \eta_{(i,j)} \tag{2b}$$

From a multiobjective programming standpoint, the preceding biobjective programming problem can be transformed into a single-objective problem by either a weighting formulation or an $\epsilon$-constraint formulation. The former leads to a relatively simple quadratic programming problem, which coincides with an ordinary linear regression model, while the latter introduces hard nonlinear constraints if the deviation is represented by the squared error. The weighted formulation is adopted to combine the two sets of deviations, with respective weights $w$ and $(1 - w)$ for the first and second objectives. The weights $w$ and $(1 - w)$ could be interpreted as the decision maker's relative preference or importance belief for the different objectives; they could also be considered as the dispersion scales for the first and second error terms in the ordinary least-squares estimation procedure. In general, if the provided target demand is not reliable—that is, the error term $\eta_{(i,j)}$ has a high variance—a small value of $w$ is used, and vice versa. The resulting bilevel dynamic O-D estimation problem with a single day of link-level observations is presented in Equations 3 and 4, which is to minimize the combined deviations, subject to the dynamic traffic assignment constraint and nonnegativity constraints for demand variables.

$$\min Z = \left\{ (1 - w) \sum_{l,h} \left[ \sum_{t,i,j} p_{(l,h),(t,i,j)} \times d_{(t,i,j)} - c_{(l,h)} \right]^2 + w \sum_{i,j} \left[ \sum_{t} d_{(t,i,j)} - g_{(i,j)} \right]^2 \right\} \tag{3}$$

subject to

$$p_{(l,h),(t,i,j)} = \text{assignment} \lfloor d_{(t,i,j)} \rfloor \qquad \text{from DTA, } \forall\, l, h, t, i, j$$

$$d_{(t,i,j)} \geq 0 \qquad \forall\, t, i, j \tag{4}$$

where $w$ is a positive weight.

If a time-dependent demand matrix is available a priori, the preceding formulation can be written as Equation 5, where $g_{(i,j)}$ is extended to $g_{(t,i,j)}$ for each departure time interval:

$$\min Z = \left\{(1-w)\sum_{l,h}\left[\sum_{t,i,j}\hat{p}_{(l,h),(t,i,j)} \times d_{(t,i,j)} - c_{(l,h)}\right]^2 + w\sum_{t,i,j}\left[d_{(t,i,j)} - g_{(t,i,j)}\right]^2\right\} \quad (5)$$

A natural attempt would be to split the given static demand $g_{(i,j)}$ into equal portions of $g_{(t,i,j)} = g_{(i,j)}/T$ for each time interval and use Equation 4, which has a structure similar to that of the static O-D estimation case. However, this scheme would implicitly impose a uniform temporal pattern on the target demand, thereby biasing the resulting estimation. More precisely, defining the combined error term $\eta_{(t,i,j)} = d_{(t,i,j)} - g_{(t,i,j)}$ for each departure time interval gives $\eta_{(i,j)} = \sum_t \eta_{(t,i,j)}$. The bias for the latter formulation with respect to the previous formulation for each O-D pair $(i, j)$ is presented in Equation 6.

$$\sum_t \left[d_{(t,i,j)} - g_{(t,i,j)}\right]^2 - \left[\sum_t d_{(t,i,j)} - g_{(i,j)}\right]^2 = \sum_t \eta^2_{(t,i,j)} - \left[\sum_t \eta_{(t,i,j)}\right]^2 = -2\sum_{t1}\sum_{t2\neq t1} \eta_{(t1,i,j)}\eta_{(t2,i,j)} \quad (6)$$

The iterative solution algorithm for the proposed bilevel programming problem is briefly described as follows. Detailed discussion can be found elsewhere (*3, 4*).

Step 1: initialization; $i = 0$. Start from an initial guess of the traffic demand matrix $D_0$, obtain link-flow proportions $P_0$ from the DTA simulator.

Step 2: optimization. Substituting link-flow proportions $P_i$, solve the dynamic O-D estimation problem as Equation 3 to obtain demand $D_i$.

Step 3: simulation. Using demand $D_i$, run the DTA simulator to generate new link-flow proportions $P_{i+1}$.

Step 4: evaluation. Calculate the deviation between simulated link flows and observed link counts, and calculate the deviation between estimated demand $D_i$ and target demand.

Step 5: convergence test. If the convergence criterion is satisfied (estimated demand is stable or no significant improvement in the overall objective), stop; otherwise $i = i + 1$ and go to Step 2.

In the following, two key questions are addressed in using the preceding formulation in planning applications. One question is how to assess the weight $w$, and the other is how to deal with estimation with partial observation.

## Retrieving Best Compromise Solution

It may be possible to obtain the least-squares estimate of the weight value through linear regression (*13*). However, in planning analysis, it is more desirable to incorporate the planners' knowledge and experience in the estimation process, reflecting different degrees of confidence in the different sources of information. Furthermore, planners might like to adjust their preferences progressively as they develop a better understanding of the problem. For these reasons, an interactive approach is presented to determine the weight for the preceding biobjective problem, consisting of the following two steps. A representative subset of nondominated solutions is first generated by varying the weight, and then the decision maker can determine the weight that results in the best compromise solution based on the following three criteria, as commonly used in the multiobjective programming field.

- Minimum combined deviation. This is equivalent to the objective function value in Equation 3.
- Best trade-off. The trade-off measurement can be computed by $\partial Z_1/\partial Z_2$, where $Z_1$ and $Z_2$ are the first and second objectives. Given two different weights $w^0$ and $w^1$, one has the corresponding objective values $Z(w^0) = [Z_1(w^0), Z_2(w^0)]$ and $Z(w^1) = [Z_1(w^1), Z_2(w^1)]$, and then $\partial Z_1/\partial Z_2$ can be numerically approximated from the ratio of change between the $Z_1$ and $Z_2$ as indicated in Equation 7. Intuitively, a trade-off in the O-D estimation problem means how much deviation from the target demand the decision maker would give up to decrease the deviation for link counts by one unit.

$$\frac{\partial Z_1}{\partial Z_2} = \frac{Z_1(w^0) - Z_1(w^1)}{Z_2(w^0) - Z_2(w^1)} \quad (7)$$

- Minimum distance from the ideal point. Planners can define the goal $f_1$ and $f_2$ as the maximum possible deviation for the first and the second objectives, and then the goals for both objectives make up an ideal point $f^* = (f_1, f_2)$ or a utopia point. The best compromise solution is the one with minimum distance from the ideal point.

## Utilizing Limited Real-Time Data

Given a subset of links with real-time link flow observations, a fundamental question is how to identify the demand dynamics with limited information. In particular, the authors are interested in obtaining a unique solution for the preceding ordinary least-squares formulation. This requires that the number of decision variables (O-D demand flows) be less than the number of constraints (the number of link observations plus the number of O-D pairs in the static demand matrix), as indicated in Inequality 8.

$$L \times H + I \times J \geq I \times J \times T \quad (8)$$

If the given link observations cannot satisfy Inequality 8, then the O-D estimation turns out to be an underdetermined problem, which can have numerous multiple solutions. To ensure the identifiability of the dynamic O-D estimation problem, the following four possible approaches can be used.

The first simple remedy is to increase the length of departure time intervals to reduce the number of decision variables, but this aggregation scheme will undermine the capability of modeling O-D demand dynamics. The second method is to shorten the length of observation time intervals to increase the number of observations. However, a short observation time interval would increase the possibility of linear correlation in the link-flow matrix $P$, which makes the estimation result unstable. In fact, to obtain a unique solution, it is still necessary to verify the rank condition—that is, the sum of rank for matrices $G$ and $C$ is greater than the number of variables. Because the coefficient vectors in matrix $G$ correspond to independent O-D demand, the rank of matrix $A$ is always $I \times J$. The link-flow proportion vector can be expressed as Equation 9.

$$p_{(l,h),(t,i,j)} = \sum_{k\in K(i,j)} \alpha_{(l,h),(t,i,j)} q_{(k,t)} \tag{9}$$

where

$\alpha_{(l,h),(t,i,j)}$ = time-dependent link-path incidence indicator,
$q_{(k,t)}$ = path flow choice probability of selecting path $k$ at the departing interval $t$, and
$K(i, j)$ = set of paths between origin $i$ and destination $j$.

Clearly, the path flow choice probability is determined by traffic assignment, and the link-path incidence is governed by the traffic flow propagation process. For instance, consider two consecutive short intervals, $t_1$ and $t_2$; it is highly possible that the path flow choice probability and the time-dependent link-path incidence are unchanged, and the corresponding two link-flow proportion vectors at $t_1$ and $t_2$ are the same. This implies that one cannot arbitrarily shrink the observation interval to increase the number of observations, and the redundant information does not increase the chance of making the problem identifiable.

For a traffic network with partial observations, not all O-D demand will pass through those links that have flow measurements. In other words, only those O-D demand flows that have an impact on the measured flows can be inferred from the observed flows. Based on the link-flow proportions generated from the network loading (simulation assignment) result, O-D pairs $(i, j)$ with $p_{(l,h),(t,i,j),m} > 0$ can be denoted as relevant O-D pairs, and those with $p_{(l,h),(t,i,j),m} = 0$ can be denoted as irrelevant O-D pairs. Consequently, only relevant O-D pairs need to enter the O-D estimation problem. However, this procedure is still an ad hoc technique that highly relies on the quality of simulated link-flow proportions, and it is still possible to rule out actual relevant O-D pairs. Therefore, one should try to estimate the full O-D matrix table as completely as possible in planning practice.

The fourth approach is to apply the polynomial transformation (*6*) presented in Equation 10. In particular, if the degree of the polynomial model $N$ is less than the number of departure intervals $T$, the number of total decision variables can be reduced. An advantage of this method is that it uses fewer decision variables to represent the dynamics of demand, especially the trend information. However, it should be noted that a low-order polynomial model may not always capture the full randomness of demand, and a high-order model might lead to wild oscillations even if it provides better goodness of fit.

$$d_{(t,i,j)} = \sum_{n=0}^{N} \beta^{n}_{(i,j)} t^{n} \tag{10}$$

where $n$ is the order term and $\beta$ is the parameter to be estimated.

## MULTIDAY (WEEKDAYS) O-D DEMAND ESTIMATION

### Model Specification

The formulation of the O-D demand estimation problem with single-day link observations is extended to a multiday context. Considering 5 weekdays, a more extensive model can be expressed as Equation 11.

$$\begin{pmatrix} (1-w)C_1 \\ (1-w)C_2 \\ (1-w)C_3 \\ (1-w)C_4 \\ (1-w)C_5 \\ wG \\ wG \\ wG \\ wG \\ wG \end{pmatrix} = \begin{bmatrix} (1-w)P_1 & 0 & 0 & 0 & 0 \\ 0 & (1-w)P_2 & 0 & 0 & 0 \\ 0 & 0 & (1-w)P_3 & 0 & 0 \\ 0 & 0 & 0 & (1-w)P_4 & 0 \\ 0 & 0 & 0 & 0 & (1-w)P_5 \\ wA & 0 & 0 & 0 & 0 \\ 0 & wA & 0 & 0 & 0 \\ 0 & 0 & wA & 0 & 0 \\ 0 & 0 & 0 & wA & 0 \\ 0 & 0 & 0 & 0 & wA \end{bmatrix} \times \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{pmatrix} + \begin{pmatrix} (1-w)E_1 \\ (1-w)E_2 \\ (1-w)E_3 \\ (1-w)E_4 \\ (1-w)E_5 \\ w\Pi_1 \\ w\Pi_2 \\ w\Pi_3 \\ w\Pi_4 \\ w\Pi_5 \end{pmatrix} \tag{11}$$

This formulation is analogous to a multiple linear regression model that has the standard form as Equation 12. From the multiple linear regression point of view, $Y$ represents dependent variables, $(X_1\ X_2\ X_3\ X_4\ X_5)$ are independent variables, $(D_1\ D_2\ D_3\ D_4\ D_5)^T$ are coefficients to be estimated, and $\Psi$ represents error terms.

$$Y = (X_1\ X_2\ X_3\ X_4\ X_5) \times \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{pmatrix} + \Psi \tag{12}$$

Three possible assumptions about $D_m$ that lead to different forms for Equation 12 are considered.

The first is that the O-D demand matrices $D_m$ on different days are different. Accordingly, link-flow proportions—that is, $X_m$'s in Equation 12—would also have different values.

The second situation corresponds to identical $D_m$ on different days. Thus, link-flow proportions over different days would be generated identically from the DTA simulation, and Equation 12 can be collapsed to the simple form presented in Equation 13.

$$Y = (X_0\ X_0\ X_0\ X_0\ X_0) \times \begin{pmatrix} D_0 \\ D_0 \\ D_0 \\ D_0 \\ D_0 \end{pmatrix} + \Psi \tag{13}$$

However, this assumption $D_m$ is likely to be too stringent for the real-world traffic demand. To recognize the inherent stochasticity of traffic demand, the third assumption views the multiday O-D demand as the outcome of a common underlying random process with mean $D_0$ and variance $\epsilon_D$—that is, $D_m = D_0 + \epsilon_D$. In this way, Equation 12 can be simplified to Equation 14.

$$Y = (X_1\ X_2\ X_3\ X_4\ X_5) \times \begin{pmatrix} D_0 \\ D_0 \\ D_0 \\ D_0 \\ D_0 \end{pmatrix} + \Psi' \tag{14}$$

where $\Psi'$ is the combined error for $\Psi$ and $\epsilon_D$.

The preceding multiday O-D estimation problem can still be solved by the bilevel ordinary least-squares method used for single-day estimation described previously. Its mathematical formulation consists of objective Function 15 and Constraint 16. Note that the objective is to minimize multiday discrepancies, and the link-flow proportions are obtained from the DTA simulator individually for different demand matrices.

$$\text{obj.}\ \min Z = \sum_m \left\{ (1-w)\sum_{l,h}\left[\sum_{t,i,j} \hat{p}_{(l,h),(t,i,j),m} \times d_{(t,i,j),m} - c_{(l,h),m}\right]^2 + w\sum_{i,j}\left[\sum_t d_{(t,i,j),m} - g_{(i,j)}\right]^2 \right\} \tag{15}$$

subject to

$$d_{(t,i,j),m} \geq 0 \qquad \forall t, i, j, m$$

$$\hat{p}_{(l,h),(t,i,j),m} = \text{assignment}\ \lfloor d_{(t,i,j),m} \rfloor \text{ from DTA} \qquad \forall\, l, h, t, i, j, m \tag{16}$$

## Analysis of Day-to-Day Variability

### Hypotheses for $D_m$

To identify day-to-day variability of O-D demand, two potential models are assumed. The null hypothesis ($H_0$) for $D_m$ is that the means of multiday demand are identical, corresponding to a reduced model. The alternative hypothesis ($H_1$) for $D_m$ is that the means of multiday demand patterns are different, corresponding to a full model.

$$H_0 : D_{m1} = D_{m2} \qquad \text{for } m_1 \neq m_2$$

$$H_1 : D_{m1} \neq D_{m2} \qquad \text{for } m_1 \neq m_2$$

### Standard F-Test

Statistical testing is performed to compare means across multiple days. The $F$-statistic tests the null hypothesis that the multiple means of O-D demands across all days are equal. If the computed $F$-statistic value is greater than the corresponding critical value (for the desired significance level), then the null hypothesis can be rejected, and the multiday mean O-D demands may be considered significantly different from one another.

$$F = \frac{(SSE_{\text{reduced}} - SSE_{\text{full}})/(k-g)}{SSE_{\text{full}}/[n-(k+1)]} \tag{17}$$

where

$n$ = number of observations,
$k$ = number of restrictions causing change of the full model to the reduced model,
$g + 1$ = number of coefficients in the reduced model, and
$SSE$ = sum of square errors, which is calculated according to Equation 18.

$$\begin{aligned} SSE &= \sum_m \left\{ (1-w)\sum_{l,h}\left[\sum_{t,i,j} \hat{p}_{(l,h),(t,i,j),m} \times \hat{d}_{(t,i,j),m} - c_{(l,h),m}\right]^2 + w\sum_{i,j}\left[\sum_t \hat{d}_{(t,i,j),m} - g_{(i,j)}\right]^2 \right\} \\ &= \sum_m SSE_m \end{aligned} \tag{18}$$

It is worth noting that the variables ($X_1, X_2, X_3, X_4, X_5$) in this study would have different values in the full model and the reduced model because of the inherent dependency of link-flow proportions and O-D traffic demand matrices.

### Full Model Calibration

Based on the structure of the model (Equation 12), the procedure for full model calibration is equivalent to performing individual model calibration for each day. After five estimated O-D demand matrices are obtained individually—that is ($\hat{D}_1, \hat{D}_2, \hat{D}_3, \hat{D}_4, \hat{D}_5$)—they just need to be substituted in Equation 18 to compute $SSE_{\text{full}}$.

### Reduced Model Calibration

The following procedure is adopted to calibrate the reduced model:

Step 1: Compute average the $\bar{D}$ and variance $\sigma_{\bar{D}}^2$ of the estimated O-D demand ($\hat{D}_1, \hat{D}_2, \hat{D}_3, \hat{D}_4, \hat{D}_5$) obtained from the full model calibration.

Step 2: Randomly generate O-D demand on five days ($\dot{D}_1, \dot{D}_2, \dot{D}_3, \dot{D}_4, \dot{D}_5$) based on the average $\bar{D}$ and the variance $\sigma_{\bar{D}}^2$.

Step 3: Obtain five link-flow proportions matrices ($\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4, \hat{P}_5$) through DTA simulation, given the randomly generated ($\dot{D}_1, \dot{D}_2, \dot{D}_3, \dot{D}_4, \dot{D}_5$).

Step 4: Substitute ($\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4, \hat{P}_5$) into Formulation 18 to estimate a common demand matrix $D_0$ for all days. Then, the estimated $D_0$ is taken as a new average value used for randomization in Step 2. If $D_0$ is stable, stop and the estimated $D_0$ in this step is the optimal one. Otherwise, go back to Step 2 and repeat the process.

Finally, $SSE_{\text{reduced}}$ can be computed based on the optimal O-D demand $D_0$.

## CASE STUDY

In this section, the proposed model and algorithm are tested by using a simplified road network of the Irvine, California, test bed, which consists of three freeway corridors (I-5, I-405, Highway 133) and other main arterials. As indicated in Figure 1, the simplified network includes 16 O-D zones, 31 nodes, and 80 links (32 freeways and 48 arterials), where traffic counts are measured on 16 links at 30-s

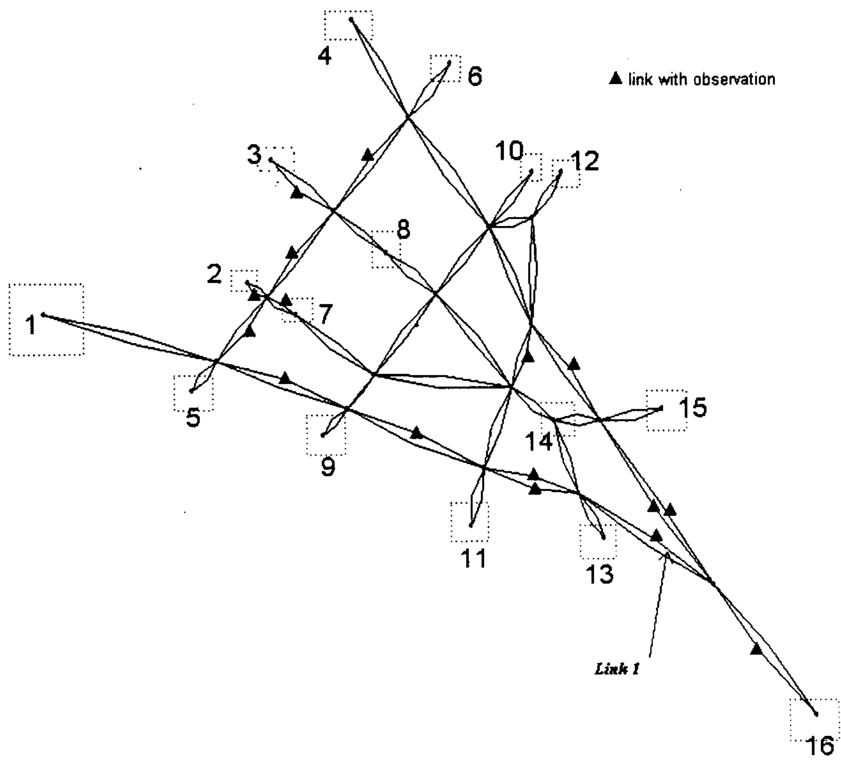


FIGURE 1 Irvine simplified network.

intervals on 10 freeway links and at 5-min intervals on 6 arterial links. In addition, a static planning O-D demand table is given and used as the target demand $g_{(i,j)}$. The time of interest in the following experiments is the morning peak period (6:30 to 8:30 a.m.) of four weekdays (Tuesday to Friday). It would have been ideal to investigate the O-D demand variability over 5 weekdays, but the data for Monday could not be used in the estimation because of poor quality (due to sensor malfunction). Simulation is performed for 3 h (6:00 to 9:00 a.m.). To gain more reliable estimation results, the starting period from 6:00 to 6:30 is used as a warm-up period. Moreover, the ending period from 8:30 to 9:00 is not considered in the statistics, because a large number of vehicles departing after 8:30 would not have finished their trips, resulting in incomplete link-flow proportions.

## Weighting Scheme in Upper-Level Optimization

Before conducting the O-D estimation, one first wants to determine the appropriate weighting value in the weighted objective function. With the data for Tuesday, 10 different values of $w$ between 0 and 1 are used to generate a representative set of nondominated solutions, as presented in Figure 2. For $w = 0$ (i.e., O-D estimation without target demand), the resulting deviation from the target demand is $2.42 \times 10^8$, which is too large to be shown in the plot. As expected, greater weight on the target demand can result in smaller deviation from the target demand but larger deviation from the link flows. Interestingly, the rate of change for the second objective is much more dramatic than for the first objective, and the total weighted deviation is governed by the deviation from the target demand. In addition, $w = 1$ minimizes the deviation from the target demand, actually yielding an overall deviation of zero, because the target demand can always be a feasible solution. In contrast, for $w = 0$ (i.e., only the deviations of link flows are considered), the solution does not perfectly fit all the link counts. The three criteria discussed earlier for determining a best compromise solution are examined here to determine the appropriate weighting value. First, the minimum combined deviation condition is not suitable in this case, because $w = 1$ always provides the best result but does not consider the link flows. It is easy to check that $w = 0.9$ corresponds to the "best" trade-off, but the absolute deviation for the link counts is still very high. The ideal point is presented in Figure 3, where the goals for the first and second objectives are set to $2.00 \times 10^6$ and 0, respectively. The plot reveals that the solutions corresponding to weights of 0.2 and 0.5 are very close to the ideal point. In this study, $w = 0.5$ is used because this provides better trade-offs.

## Day-to-Day O-D Demand Patterns

The estimated demand patterns for three O-D pairs are presented in Figures 4 to 6. In particular, the selected O-D pairs (12, 1), (16, 1), and (16, 4) are representative O-D pairs with the highest trip demand. In Figure 4, the estimated dynamic demand on different days is consistent with the magnitude of the historical static demand. As expected, all three O-D pairs show significant within-day dynamics. Particularly, O-D pair (12, 1) corresponds to a slow increasing trend, and O-D pairs (16, 1) and (16, 4) have similar peak patterns: the peak occurs around 7:15 a.m., and the height of the peak is 20% to 30% above the minimal demand level during the 2 h. Note that the latter two O-D pairs start from the same origin Zone 16, so they are more likely to have common departure time patterns. Moreover, the dynamic demand pattern can be verified by the observed link flows as indicated in Figure 7, observed versus simulated link flows for Link 1. The reason for selecting Link 1 (presented in Figure 1) is that it carries the demand flow for O-D pair (16, 1). It is easy to see that the time of flow

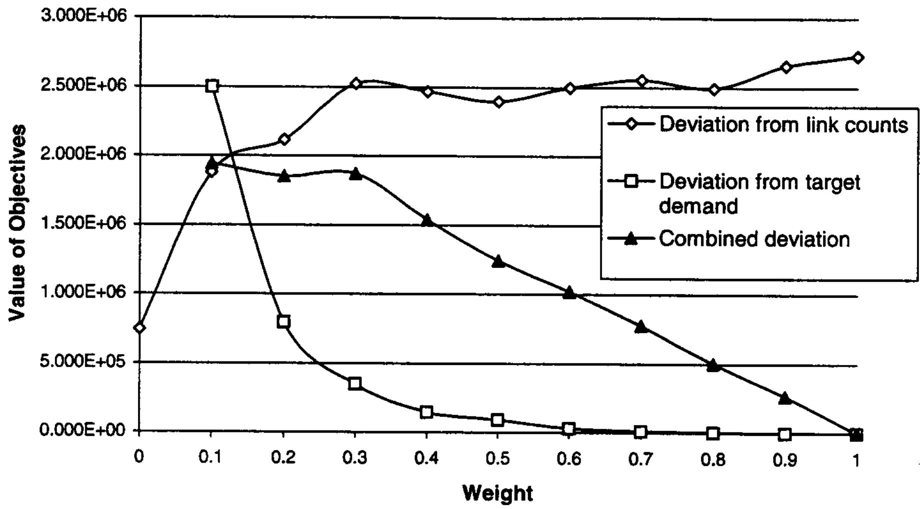


FIGURE 2 Representative set of nondominated solution generated by weighted method.

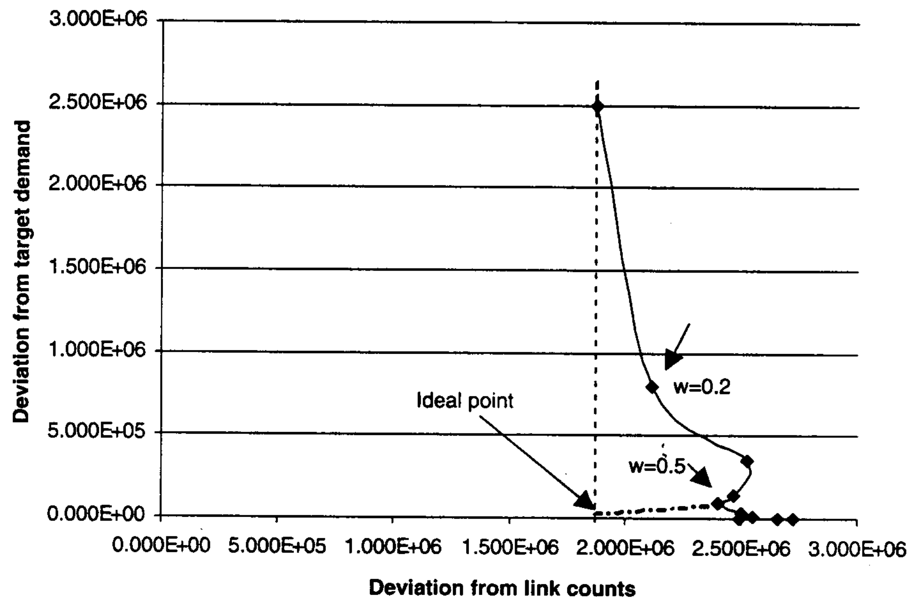


FIGURE 3 Illustration for ideal point.

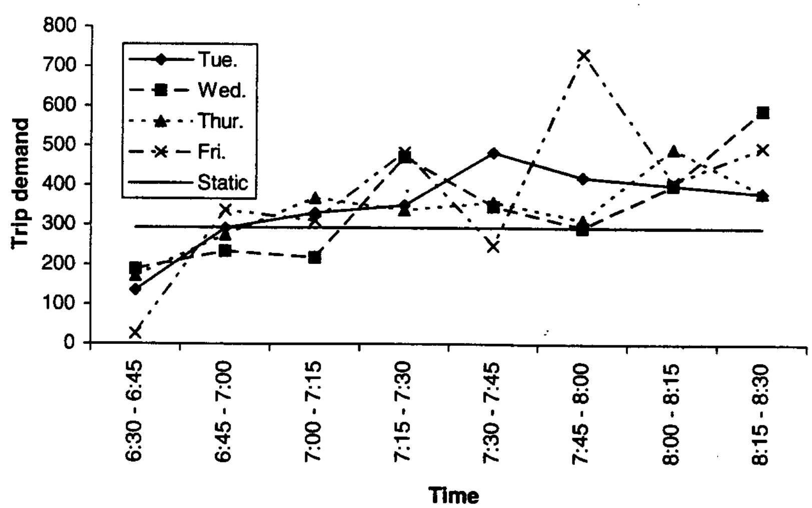


FIGURE 4 Estimated trip demand patterns for O-D pair (12, 1).

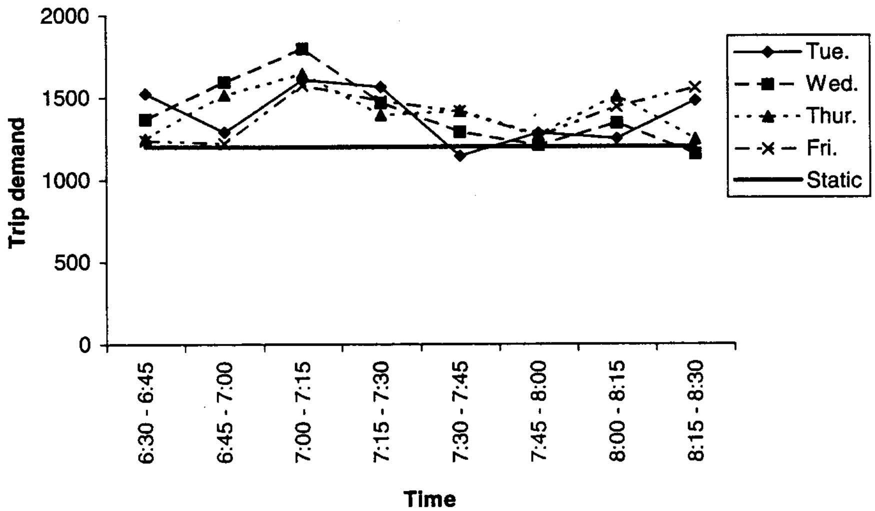


FIGURE 5 Estimated trip demand patterns for O-D pair (16, 1).

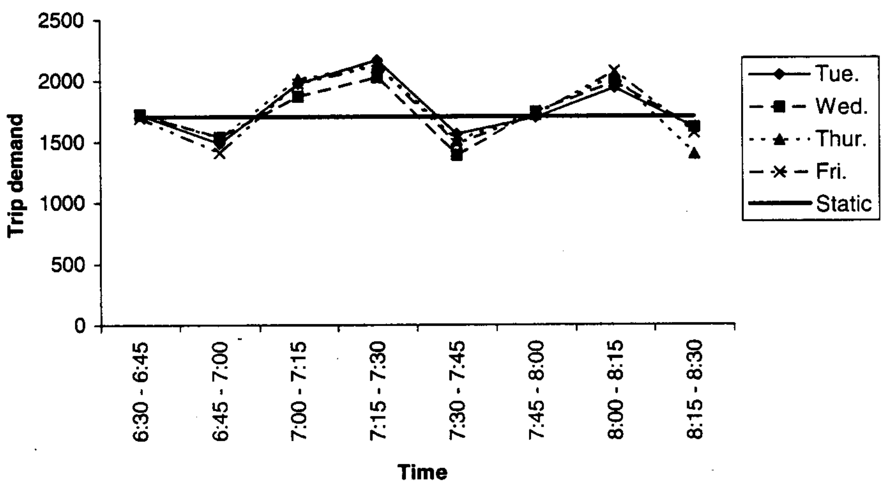


FIGURE 6 Estimated trip demand patterns for O-D pair (16, 4).

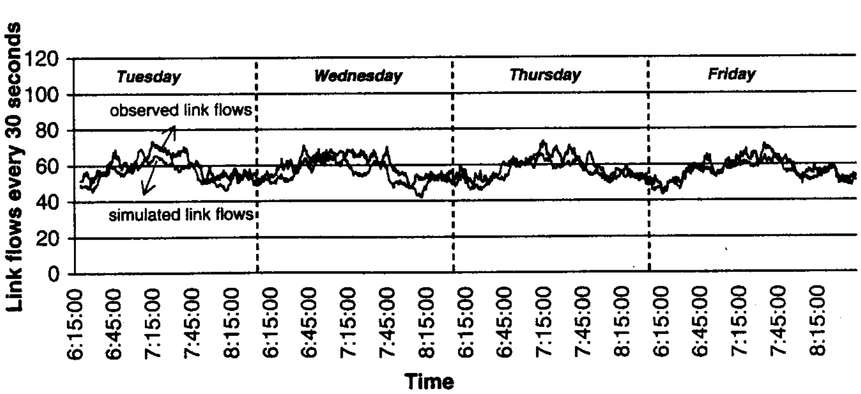


FIGURE 7 Observed versus simulated link flows for Link 1.

peak is relatively later than the time of demand peak, which is around 7:15 to 7:45 a.m., due to the traffic flow propagation. Furthermore, on Link 1, the simulated flows match the observed flows quite well.

Based on the estimated results, all three O-D pairs exhibit different patterns of day-to-day variation. In general, O-D pair (12, 1) exhibits greater variation in terms of the time of peak, O-D pair (16, 1) shows greater variation in terms of the height of peak, while O-D pair (16, 4) exhibits much more stable patterns. This information is useful to help the transportation manager alleviate congestion by redistributing demand flow spatially and temporally on the network.

### Hypothesis Testing for Mean of Demand

As indicated in Equation 17, the sum of squared errors $SSE$ for estimation (say, the total objective value) is required for computation of the $F$-statistic. Calibration of the full model and the reduced model provide the value of $SSE$ without additional effort. The results are $SSE_{\text{full}} = 4.90 \times 10^6$ for the full model and $SSE_{\text{reduced}} = 5.55 \times 10^6$ for the reduced model. In addition, during 2 h (or 120 min) and for $16 \times 16 - 16 = 240$ O-D pairs, the number of observations $n = (120 \times 2 \times 10 + 120/5 \times 6 + 240) \times 4 = 11{,}136$. The number of restrictions from the full model to the reduced model is $k = 120/15 \times 240 \times 3 = 5{,}760$. The number of coefficients in the reduced model is $g + 1 = 120/15 \times 240 = 1{,}920$. The significance level $\alpha$ is set to 0.05. Therefore, the $F$-statistic is calculated as follows:

$$F = \frac{(SSE_{\text{reduced}} - SSE_{\text{full}})/(k - g)}{SSE_{\text{full}}/[n - (k + 1)]} = 0.186$$

Comparing this value with the critical value, $F_{\alpha,k-g,n-(k+1)} = 1.10$, the null hypothesis that the restriction is valid stands, indicating that the mean O-D demand pattern for the network is essentially identical across multiple days.

Because link-flow proportions are generated by simulation instead of known a priori, performing an $F$-test for the model specification in this example should be viewed as approximate and interpreted with caution. Furthermore, O-D demands might not be independent of each other, so the conclusion from the $F$-statistic again should be interpreted with considerable caution.

## CONCLUSION

Time-dependent O-D demand matrices are a critical input to dynamic traffic assignment methodology in real-time operational and planning applications. This paper introduces and highlights the potential of using multiple sources of information to estimate the dynamic O-D demand for planning applications. The particular sources available here include the historical static information and ITS real-time link-level information. First, a bilevel iterative dynamic O-D estimation model is extended to combine both deviation between estimated link flows and real-time link counts and deviation between estimated time-dependent demand and given historical static demand. The two objectives are combined by using a weighted function, where the weighting value is determined by an interactive approach to obtain the best compromise solution. In particular, the trade-offs among several methods that are designed to use limited real-time information to infer the demand dynamics are discussed. The model was extended to use the multiple days of link counts to estimate the variations in traffic demand over multiple days. The case study based on the Irvine network illustrates that valuable information about traffic demand dynamics can be estimated with multiday data.

Future research in dynamic O-D demand estimation for planning applications includes using other ITS information sources such as automatic vehicle identification and GPS as well as other static information such as peak hour counts and daily counts. It is also desirable to use multiple weeks of data in demand estimation and integrate with a disaggregate analysis for day-to-day demand dynamics to improve the final quality of O-D estimation.

## ACKNOWLEDGMENTS

This paper is based in part on research funded through the Southwest University Transportation Center, the Texas Department of Transportation, and the U.S. Department of Transportation. The work has benefited from the contributions of several current and former graduate students, including Hossein Tavana, Yi-Chang Chiu, and Hayssam Sbayti.

## REFERENCES

1. Cremer, M., and H. Keller. Dynamic Identification of Flows from Traffic Counts at Complex Intersections. *Proc., 8th International Symposium on Transportation and Traffic Theory*, Toronto University, Ontario, Canada, 1981.
2. Cascetta, E., D. Inaudi, and G. Marquis. Dynamic Estimators of Origin-Destination Matrices Using Traffic Counts. *Transportation Science*, Vol. 27, No. 4, 1993, pp. 363–373.
3. Mahmassani H. S., and H. Tavana. Estimation of Dynamic Origin-Destination Flows from Sensor Data Using Bilevel Optimization Method. Presented at 80th Annual Meeting of the Transportation Research Board, Washington, D.C., 2001.
4. Tavana, H. *Internally Consistent Estimation of Dynamic Network Origin–Destination Flows from Intelligent Transportation Systems Data Using Bi-Level Optimization.* Ph.D. dissertation. University of Texas at Austin, 2001.
5. Peeta, S., and A. Ziliaskopoulos. Foundations of Dynamic Traffic Assignment: The Past, the Present and the Future. *Networks and Spatial Economics*, Vol. 1, No. 3/4, 2001, pp. 233–266.
6. Kang, Y. *Estimation and Prediction of Dynamic Origin–Destination (O-D) Demand and System Consistency Control for Real-time Dynamic Traffic Assignment Operation.* Ph.D. dissertation. University of Texas at Austin, 1999.
7. Chang, G. L., and X. D. Tao. An Integrated Model for Estimating Time-Varying Network Origin–Destination Distributions. *Transportation Research A*, Vol. 33, No. 5, 1999, pp. 381–399.
8. Ashok, K., and M. Ben-Akiva. Alternative Approaches for Real-Time Estimation and Prediction of Time–Dependent Origin–Destination Flows. *Transportation Science*, Vol. 34, No. 1, 2000, pp. 21–36.
9. Waller, S. T., and A. Ziliaskopoulos. A Chance-Constrained Based Stochastic Dynamic Traffic Assignment Model: Analysis, Formulation and a Solution Algorithm. *TRISTAN III Proceedings*, San Juan, Puerto Rico, 1998.
10. Koppelman, F. S., and E. I. Pas. Estimation of Disaggregate Regression Models of Person Trip Generation with Multiday Data. *Proc., 9th International Symposium on Transportation and Traffic Theory.* VNU Science Press, Utrecht, Netherlands, 1984, pp. 513–529.
11. Yalamanchili, L., R. M. Pendyala, N. Prabaharan, and P. Chakravarthy. Analysis of Global Positioning System–Based Data Collection Methods for Capturing Multistop Trip-Chaining Behavior. In *Transportation Research Record: Journal of the Transportation Research Board, No. 1660*, TRB, National Research Council, Washington, D.C., 1999, pp. 58–65.
12. Mahmassani, H. S., et al. *DYNASMART-P, Volume II, User's Guide.* Technical Report STO67-85-PII. Center for Transportation Research, University of Texas at Austin, 2000.
13. Cascetta, E., and S. Nguyen. A Unified Framework for Estimating or Updating Origin/Destination Matrices from Traffic Counts. *Transportation Research B*, Vol. 22, 1988, pp. 437–455.

*The authors are responsible for all results and opinions expressed in this paper.*

*Publication of this paper sponsored by Committee on Passenger Travel Demand Forecasting.*